A Variational Technique for Dealiasing Doppler Radial Velocity Data

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13 July 2003 and 29 November 2003

ABSTRACT

Velocity folding, or aliasing, is one of most significant impediments to the use of radial winds from Doppler weather radar. In this note, a variational algorithm is developed in which dealiasing is performed using wind gradient information. The key to the proposed method is that, by operating on gradients of velocity rather than on the velocity itself, aliasing ambiguities are readily identified and eliminated. The viability of the method is demonstrated by applying it to Weather Surveillance Radar-1988 Doppler (WSR-88D) observations from a winter-weather event and a tornadic supercell storm.

1. Introduction

The successful utilization of Doppler weather radar data in numerical prediction models, hazard detection algorithms, and decision support systems is heavily dependent upon data quality. The most serious sources of data contamination are velocity ambiguities, range ambiguities, and ground clutter associated with normal and anomalous propagation (Serafin and Wilson 2000). Velocity ambiguities occur when the true velocity exceeds the unambiguous (Nyquist) velocity, and the removal of such errors is vitally important before the data are used in a quantitative manner (e.g., Eilts and Smith 1990). For example, our experiments indicate that if only 3% of a full volume scan of radial wind data are contaminated by aliasing errors, relative rms errors in the mean vertical wind profile retrieved using the velocity azimuth display (VAD; Browning and Wexler 1968) technique can reach 50% (see also Gao et al. 2004).

Since the 1970s, several methods have been developed to dealias Doppler radial velocity data. Ray and Ziegler (1977) developed a one-dimensional dealiasing technique that required range gates along a radial to be normally distributed about their mean. The method worked only when minor aliasing errors were present. Another one-dimensional method, developed by Bargen and Brown (1980), used spatial continuity of the wind field along each radial to remove local aliasing. They assumed that the first range gate in each radial was free of error. The lack of generality of this assumption leads to its frequent violation, and thus the method typically requires human intervention.

Merritt (1984) proposed a potentially more powerful approach by using data in two dimensions, that is, by searching for aliasing errors along both the radial and azimuthal directions in each sweep of the volume scan. Bergen and Albers (1988) expanded upon it by using Weather Surveillance Radar-1988 Doppler (WSR-88D) VAD wind profiles to include mean wind information. They found that VADs were adequate to resolve isolated regions of aliasing; however, VAD winds themselves also are very sensitive to aliasing errors, thus leading to a circular problem. A modified VAD technique developed by Tabary et al. (2001), in which the mean wind profile obtained is reliable and free of aliasing errors, essentially eliminates this problem. Jing and Wiener (1993) developed a sophisticated two-dimensional algorithm that solves a linear system to minimize gate-to-gate shear in each isolated echo. They assumed that the averaged radar wind is less than the Nyquist velocity, with heavy reliance upon the accuracy of an environmental wind field estimate or VAD profile.

Eilts and Smith (1990) developed the operational WSR-88D dealiasing algorithm that combines the ad-

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vantages of methods previously described. It is two-dimensional and uses a nearby vertical sounding as a background against which winds are checked for aliasing. James and Houze (2001) recently developed a new real-time four-dimensional Doppler dealiasing method that uses all available data dimensionality to interpret and dealias each tilt in real time. They applied it to C-band Doppler velocity measurements from the Swiss Monte Lema radar throughout the 2-month duration of the 1999 Mesoscale Alpine Program (MAP; Bougeault et al. 2001). The algorithm produced promising results in a variety of meteorological conditions; however, it requires six steps, including initial dealiasing, spatial dealiasing, window dealiasing, and auxiliary dealiasing, thus making the algorithm complicated. As pointed out by the authors, the method also has some shortcomings, including sensitivity to shear parameters and requirement of a continuous full data volume.

Most studies in Doppler velocity dealiasing during the past 30 yr have used various combinations of reference/background checking (e.g., against a sounding or VAD wind profile) and spatial continuity checking along the radial and azimuthal directions. Although these methods generally have been successful in most weather situations, they suffer shortcomings in extreme weather conditions and come with increased complexity. For example, reference checking suffers degradation when the horizontal shear is strong. In continuity checking, errors in the first range gate and/or the starting radial may propagate along the radial and azimuthal directions, and it is difficult to dealias data in isolated areas. Thus, in especially strong storm circulations or for low Nyquist velocities, most of the dealiasing methods fail for some gates or even for entire sectors of data.

2. Proposed method

Although velocity dealiasing may be viewed as a quality-control procedure, it more properly lies within the realm of data assimilation in that its solution can be thought of as an integral part of the data-assimilation process itself. Data assimilation typically utilizes information from background fields and observations, and in our proposed method the observation component consists of *gradients* of radial velocity in both the azimuthal and radial directions, as calculated directly from measured radial velocity. The background term comprises a combination of wind profiles obtained from a modified VAD method (Tabary et al. 2001; Gao et al. 2004) and a previous storm-scale forecast or analysis.

The key to Doppler radial velocity dealiasing and the mechanism for its inclusion in variational assimilation both lie in the fact that *gradients* of radial velocity in both the azimuthal and radial directions are *not aliased*, except in regions where aliased values adjoin nonaliased values. In such regions, which constitute only a tiny fraction of the total number of gates in a tilt, the computed gradient is orders of magnitude larger than in other regions and easily can be detected and rejected.

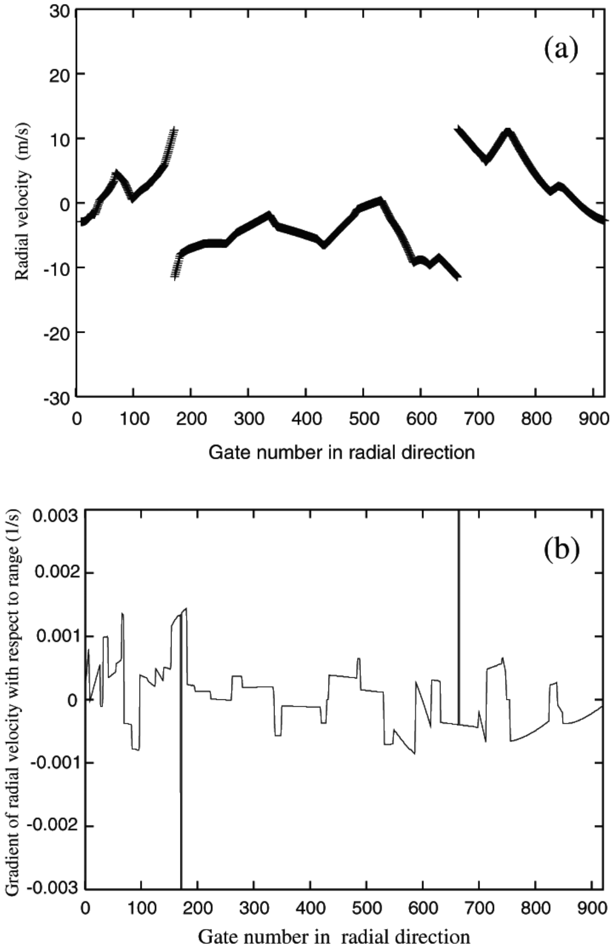

FIG. 1. (a) Idealized radial velocity (m s^{-1}) as a function of range gate (zero indicates the location of the radar) for a Nyquist interval of 11.5 m s^{-1}. Over one-half of the data (middle of plot) are contaminated by velocity aliasing. (b) Gradient of radial velocity with respect to range calculated from the idealized data in (a). The two locations for which very large values exist represent regions where aliased data adjoin nonaliased data.

To illustrate, Fig. 1a shows an idealized radial wind field, for a particular azimuth, as a function of gate number in the radial direction. The Nyquist velocity is 11.5 m s^{-1}. Over one-half of the total data in the middle part of the scan are contaminated by velocity aliasing. However, the radial gradients of the radial wind, shown in Fig. 1b, are everywhere continuous, except for the 2 out of 920 gates for which aliased radial wind data adjoin nonaliased data. These two gates contain gradient values that are several orders of magnitude higher than others in the scan and are easily detected and discarded.

Tabary et al. (2001) used gradients of radial velocity with respect to azimuth to obtain the mean wind profile in a modified VAD technique without first dealiasing the radial data. Their method consists of noting that, at a given level z_0, the first-order VAD expansion of the radial velocity may be written as

$$V_r = \left[\frac{1}{2} r \cos^2\alpha \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) + (w_0 - w_t)\sin\alpha\right] + (v_0 \cos\alpha)\cos\theta + (u_0 \cos\alpha)\sin\theta, \tag{1}$$

where α is the elevation angle; θ is the azimuthal angle; u_0, v_0, and w_0 are the Cartesian wind components; and $[(\partial u/\partial x) + (\partial v/\partial y)]$ is horizontal wind divergence at level z_0. Taking the azimuthal derivative of the above leads to

$$\partial V_r/\partial\theta = -(v_0 \cos\alpha)\sin\theta + (u_0 \cos\alpha)\cos\theta. \tag{2}$$

Thus, if azimuthal gradients $(\partial V_r/\partial\theta)$ are estimated at each radar gate, the mean horizontal wind (u_0, v_0) is easily obtained with a least squares fit by using (2). In Tabary et al. (2001), the azimuthal gradients are calculated through an additional linear fitting step using 35–50 points of radial velocity observations. However, our recent tests indicate that azimuthal gradients can be estimated reliably from two adjoining radial velocities along a particular azimuth (Gao et al. 2004). Although the calculated azimuthal gradients may contain large random errors because of instrument noise, such noise can be significantly reduced by a low-pass filter, yielding a reliable mean wind (u_0, v_0).

The above method represents a robust and computationally efficient technique for obtaining reliable vertical wind profiles. By using such profiles, the dealiasing process can be self-consistent; however, it is necessary to include the wind profile derived from the result of a previous data assimilation cycle, or other observation for situations in which valid radar data coverage is very limited.

After obtaining gradients of radial velocity and the vertical wind profile, we may solve the dealiasing problem by applying a simple variational method in which a cost function is defined as

$$\begin{aligned} J = &\iint_\Omega (V_r - V_r^{\mathrm{bg}})^2 r\, dr\, d\theta \\ &+ w_1 \iint_\Omega \frac{1}{r}\left[\frac{\partial V_r}{\partial\theta} - \left(\frac{\partial V_r}{\partial\theta}\right)^{\mathrm{ob}}\right]^2 r\, dr\, d\theta \\ &+ w_2 \iint_\Omega \left[\frac{\partial V_r}{\partial r} - \left(\frac{\partial V_r}{\partial r}\right)^{\mathrm{ob}}\right]^2 r\, dr\, d\theta \\ &+ w_3 \iint_\Omega (\nabla^2 V_r)^2 r\, dr\, d\theta, \end{aligned} \tag{3}$$

where $\int\int_\Omega () \, r\, dr\, d\theta$ stands for an integral over one tilt of a radar volume scan. The control variable is the analyzed radial velocity V_r, with V_r^{bg} being the background of radial velocity derived from a vertical wind profile obtained by the method discussed above. Because the wind field is assumed to vary linearly in this modified VAD method, the derived background of radial velocity also is linear. The quantities $(\partial V_r/\partial\theta)^{\mathrm{ob}}$ and $(\partial V_r/\partial r)^{\mathrm{ob}}$ are calculated directly from raw radial velocity data except for regions where, because of aliasing error, extremely large values are discarded. The quantities w_1, w_2, and w_3 are weights to be determined by the order of magnitude of each term.

The cost function (3) includes three terms: (a) the background radial velocity derived from a wind profile, as described above, which will be used to provide a constraint for reference; (b) the gradients of observed radial velocity with respect to range and azimuth; and (c) a smoothness constraint to reduce errors in radial velocity that might be caused by ground clutter or other artifacts.

From a procedural point of view, the variational dealiasing problem is as follows.

1) Use the modified VAD technique to calculate a vertical wind profile. If the VAD wind is not available, choose a nearby profiler observation or rawinsonde observation or extract a profile from a forecast model.
2) After obtaining the background radial velocity from the VAD or other wind profile, calculate the cost function J using (3) and its corresponding gradients $(\partial J/\partial V_r)$.
3) Use a conjugate gradient or quasi-Newton minimization algorithm (Navon and Legler 1987) to obtain updated values of the control variables,

$$V_r^{(n)} = V_r^{(n-1)} + \beta f(\partial J/\partial V_r), \tag{4}$$

where n is the number of iterations, β is the optimal step size obtained by the so-called line-search process in optimal control theory (Gill et al. 1981), and $f(\partial J/\partial V_r)$ is the optimal descent direction obtained by combining the gradients from several previous iterations.
4) Check whether the optimal solution has been found by computing the norm of the gradients or the value of J to see if they are less than a prescribed tolerance (in this case, $J_k/J_1 \leq 10^{-2}$, where J_k is cost function for the kth iteration and J_1 is the cost function for the first iteration) or if a specified maximum iteration number has been reached (we use a maximum of 50 iterations). If either criterion is satisfied, stop iterating and output the calculated radial velocities.
5) If the convergence criterion is not satisfied, repeat steps 2–4 using updated values of radial velocity as the new guess and continue the iteration until convergence is achieved.

To apply the method to severe-weather situations, such as mesocyclones and tornadoes, the weighting for the smoothing term should be kept very small in comparison with other constraints. Because no dynamic equation is included, the solution to the variational prob-

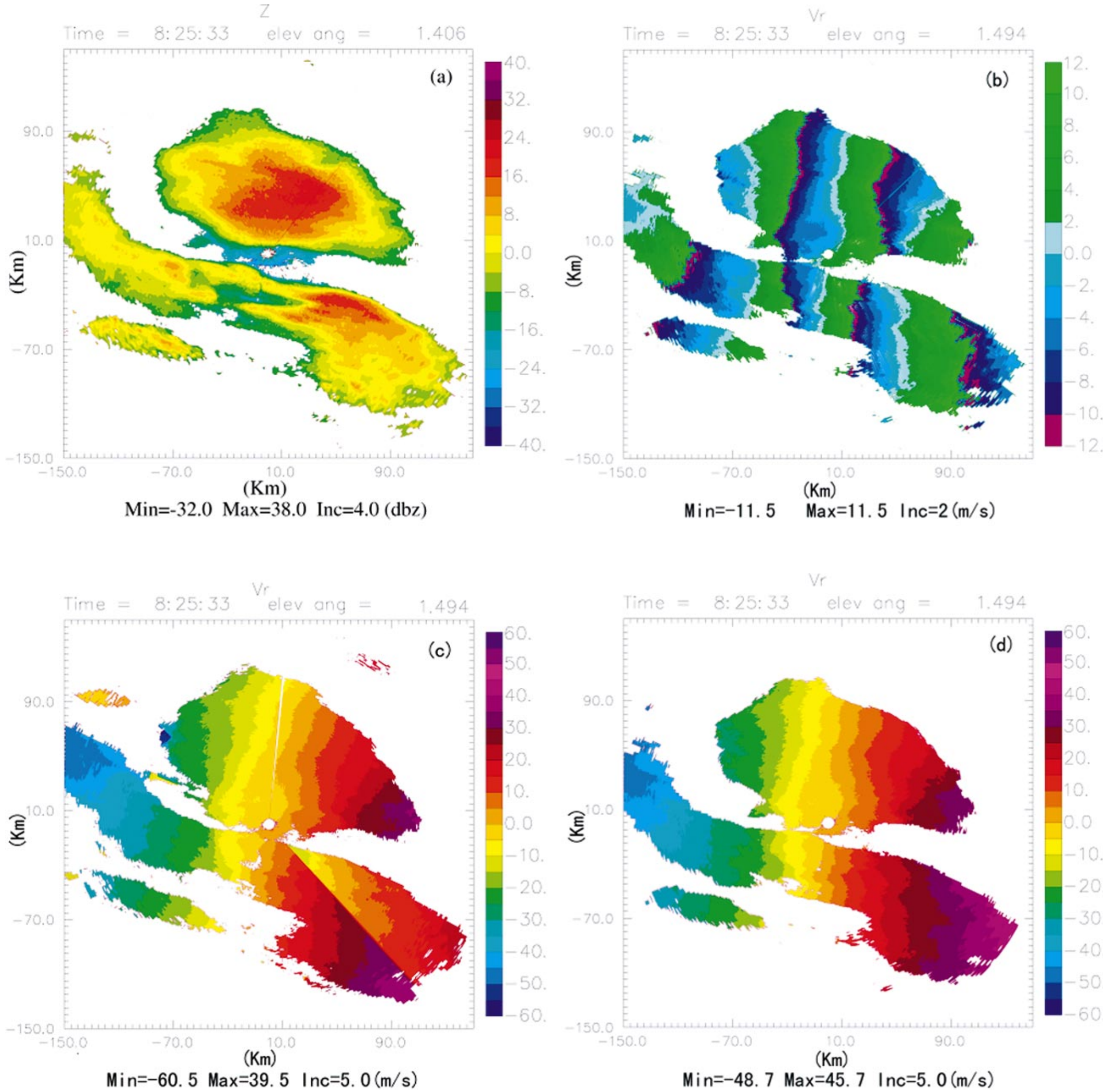

FIG. 2. (a) Reflectivity and (b) radial velocity observations for a winter storm observed by the KDTX WSR-88D near Detroit, MI, on 2 Dec 2002 for an elevation angle of 1.5°. The radar was operating in clear-air mode with a Nyquist velocity of 11.5 m s^{-1}. The radial velocity unfolded (c) using the operational WSR-88D dealiasing algorithm and (d) using the variational method developed herein.

lem can be obtained very quickly, making it especially appropriate for real-time operations. Another advantage of this method is that one need not know in which Nyquist interval the data reside in defining the cost function.

3. Preliminary results

a. Winter-storm event

To demonstrate the capability of our variational method, we apply it to several volume scans of WSR-88D data for a winter-storm event near Detroit, Michigan, on 2 December 2002. Figures 2a,b show one elevation angle of reflectivity and Doppler radial velocity observed by the KDTX (Detroit) radar at 0825 UTC. During that time, the radar was operating in clear-air mode (one of four modes used to survey the atmosphere) with a Nyquist velocity of 11.5 m s^{-1}. Because the actual winds approached 50 m s^{-1}, very severe aliasing errors, with multiple folding, are clearly evident in Fig. 2b.

Figure 2c shows the dealiased wind field produced by the operational WSR-88D dealiasing algorithm. It

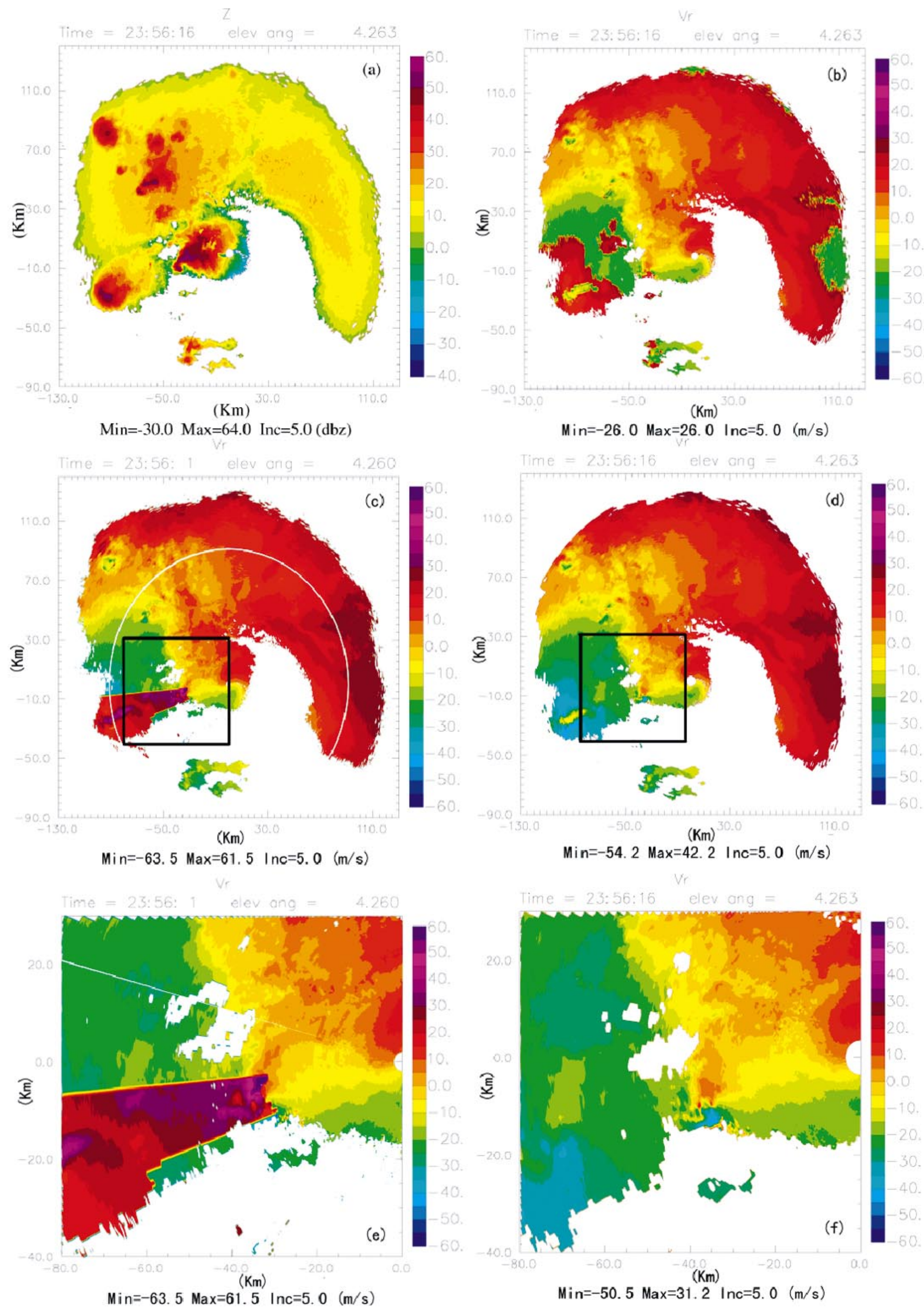

FIG. 3. As in Fig. 2, but for the 3 May 1999 tornadic storm event observed by the KTLX WSR-88D near Oklahoma City, OK, operating in precipitation/storm mode with a Nyquist velocity of 26.0 m s^{-1}. (e), (f) The enlarged images from the boxes of (c) and (d), respectively.

fails to unfold properly the velocities in the southeastern sector of the tilt (note the straight line in the southeast quadrant of the scan) because of error propagation outward along individual radials. However, the variational method (Fig. 2d) avoids spreading error from one point to an entire sector of radar data because the complicated and sometimes iterative 2D continuity checking in both azimuthal and radial directions is not used. Indeed, it unambiguously corrects all aliasing errors, including those in areas for which multiple folding was present (cf. Fig. 2b).

b. *Tornadic supercell*

To test further the performance of the variational method, we apply it to several volume scans of WSR-88D data from the 3 May 1999 tornado outbreak in central Oklahoma. On that day, several supercell thunderstorms occurred in Oklahoma, and violent tornadoes produced considerable damage. This case is very challenging because radial velocity measurements were severely contaminated by large aliasing errors, especially in the vicinity of observed mesocyclones and tornadoes.

Figures 3a,b show the lowest elevation of reflectivity and Doppler radial velocity observed by the KTLX radar (near Oklahoma City) at 2356 UTC. During that time, the radar was operating in precipitation or storm mode with a Nyquist velocity of 26 m s^{-1}. Because the actual wind approached 50 m s^{-1}, very severe aliasing errors were present in many locations, especially in the southwestern sector of the scan (red region within which is embedded folded velocities shown in yellow), where a very strong mesocyclone produced an F5 tornado (on the Fujita scale) located roughly at coordinate (-35.0, -10.0) in the plot. Other aliasing is evident in the extreme eastern part of the plot, where a patch of aliased radial velocities (green) are embedded within a large region of outbound (red) flow.

Figures 3c,d show the dealiased wind field as determined by the operational WSR-88D algorithm and our method, respectively. Aliasing errors in the far eastern part of the scan are corrected by both methods; however, the operational method fails to unfold properly the velocities in the southwestern sector of tilt, most likely because of error propagation outward from the location of the mesocyclone (Fig. 3e). The variational method corrects all aliasing errors, including those in the vicinity of the mesocyclone (Fig. 3f). Though we show here only one tilt, the method was applied successfully to many tilts and time levels of radial wind data.

4. Summary and discussion

We developed in this note a variational algorithm in which dealiasing is performed using wind gradient information. The cost function includes a background wind field for use as a reference constraint, the gradients of observed radial velocity with respect to range and azimuth for use as observational constraints, and a smoothness constraint to reduce errors caused by ground clutter or other artifacts. The key to the proposed method is that, by operating on gradients of radial velocity rather than on the velocity itself, aliasing ambiguities are readily identified and eliminated. We demonstrated the viability of the method by applying it to WSR-88D observations of a winter-storm event and a tornadic supercell storm.

In future work we will compare results from the variational technique with several existing velocity dealiasing algorithms, with emphasis on efficiency, accuracy, and robustness in a variety of weather situations. We believe that the variational method has the potential to improve substantially the velocity unfolding capability of the operational WSR-88D and, thus, the quality of products derived from it.

One of the inherent characteristics of Doppler radar is the so-called Doppler dilemma in which increasing the maximum unambiguous velocity by using a short pulse-repetition time (PRT) decreases the maximum unambiguous range, and vice versa (Rinehart 1991). The effect of this relationship is greater for short-wavelength radars such as the WSR-88D (with a wavelength of 10 cm). Mitigating the effects of the Doppler dilemma has been and continues to be a major concern of the Next-Generation Weather Radar (NEXRAD) program. This problem can obscure severe-storm signatures and clear-air phenomena (Crum et al. 1998). Because range ambiguities contaminate both the reflectivity and radial wind data, they are more difficult to correct. If a robust velocity dealiasing method is developed, the Nyquist velocity can be small even when the wind is very large, and, as a result, one can use a long PRT when operating the WSR-88D so that range-ambiguous returns can be effectively minimized.

Acknowledgments. This research was supported by NSF Grants ATM-0331756, ATM-0129892, and EEC-0313747, Grant DOT-FAA NA17RJ1227-01, and Grant ATM 99-81130 (to the second author). The authors have benefited from many discussions with Drs. Keith Brewster, Jiandong Gong, Soen-Yong Lee, Shun Liu, Qin Xu, and Pengfei Zhang and with Mrs. Janelle Janish.

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