

**METR 4433 – Mesoscale Meteorology
Spring 2017**

**Problem Set #5
Deep Convective Storms – Part II**

Distributed Saturday, 15 April 2017
Due Tuesday, 2 May 2017

INSTRUCTIONS: Please answer each of the questions shown below. Pay close attention to neatness and describe your work at each step of the solution process.

1. Storm-Relative Environmental Helicity. The storm-relative environment helicity in the lowest 3 km layer is given as

$$SREH = \int_{0km}^{3km} [(\vec{V} - \vec{C}) \cdot \vec{\omega}_H] dz$$

which, based on the definition of the horizontal vorticity vector,

$$\vec{\omega}_H = \hat{k} \times \frac{d\vec{V}}{dz} = -\frac{dv}{dz} \hat{i} + \frac{du}{dz} \hat{j},$$

can be rewritten as

$$SREH = -\int_{0km}^{3km} \hat{k} \cdot \left[(\vec{V} - \vec{C}) \times \frac{d\vec{V}}{dz} \right] dz = -\int_{0km}^{3km} \hat{k} \cdot [(\vec{V} - \vec{C}) \times d\vec{V}] = -\int_{0km}^{3km} \hat{k} \cdot [\vec{V}_r \times d\vec{V}]$$

where $\vec{V}_r \equiv (\vec{V} - \vec{C})$ is the storm-relative velocity.

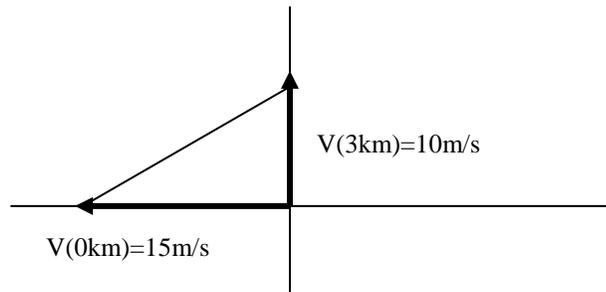
a). Using the above information and your knowledge of analytic geometry, show that the SREH is equal to minus twice the signed (i.e., positive or negative) area swept out by the storm-relative wind vector between 0 and 3 km on a hodograph. Note that, by convention, an area is positive (negative) if it is swept out counterclockwise (clockwise). To keep the problem simple, assume that wind observations are available at the 0 and 3 km levels only.

b). If the storm-relative velocity at 0 and 3 km levels is (u_{r1}, v_{r1}) and (u_{r2}, v_{r2}) , respectively, show that SREH can be calculated as

$$SERH = u_{r2}v_{r1} - u_{r1}v_{r2}.$$

Hint: See Eq. (8.15) in the text. Also, $d\vec{V} = d\vec{V}_r$ because the storm motion vector is constant with height.

c). Verify that for the hodograph shown below, and a zero storm-motion vector, that the above two methods for computing the SREH give the same results.



d). Explain why larger SERH tends to promote longer lasting supercell storms.

2. Wind Hodographs. A vertical wind profile of the horizontal is given in the following table:

<u>z (height, km)</u>	<u>θ (wind direction, deg)</u>	<u>V(speed, m/s)</u>
0	110	6
1	150	10
2	180	15
3	190	17
4	250	25
5	270	30
6	310	40

Assume that the storm motion vector is 225 degrees (from SW) at 12 m/s.

- Plot the hodograph and the storm-relative velocity vectors at each level.
- Calculate the horizontal vorticity (vector, in terms of the vorticity components or in magnitude and direction) in each of the six layers between the levels of observations.
- Determine the mean (storm-relative) wind vector in each of the six layers.
- Using the layer-mean wind obtained above, calculate the storm-relative environmental helicity in each of the six layers and determine the vertically integrated environmental helicity in the lowest three kilometers.
- Discuss your results and their significance in terms of their effect on the behavior and type of storms that occur in such an environment
- For this wind profile, what kind of CAPE values will give you a BRN that suggests a high probability of multicell and supercell storms, respectively?

3. Vorticity Equation in 3D. Consider the Boussinesq equations of motion (i.e., density is constant) neglecting both friction and the Coriolis force:

$$\frac{du}{dt} = -\frac{1}{\rho_o} \frac{\partial p'}{\partial x}$$

$$\frac{dv}{dt} = -\frac{1}{\rho_o} \frac{\partial p'}{\partial y}$$

$$\frac{dw}{dt} = -\frac{1}{\rho_o} \frac{\partial p'}{\partial z} + B$$

Where B is the buoyancy term and the prime indicates the perturbation with respect to a homogeneous base state. In vector form, these three equations may be written

$$\frac{\partial \vec{V}}{\partial t} = -\vec{V} \cdot \nabla \vec{V} - \frac{1}{\rho_o} \nabla p' + B \hat{k}.$$

(a). Using the vector identity $\vec{V} \cdot \nabla \vec{V} = \nabla \left(\frac{\vec{V} \cdot \vec{V}}{2} \right) - \vec{V} \times (\nabla \times \vec{V})$, derive the three-

dimensional vector vorticity equation $\frac{\partial \vec{\omega}}{\partial t} = \nabla \times (\vec{V} \times \vec{\omega}) + \nabla \times (B \hat{k})$, where $\vec{\omega} = \nabla \times \vec{V}$.

4. Helicity. Supercell storms are relatively long-lived and resilient to turbulent dissipation than their single cell and multicell counterparts. Consider the vector vorticity equation that you derived in Problem #2 above:

$$\frac{\partial \vec{\omega}}{\partial t} = \nabla \times (\vec{V} \times \vec{\omega}) + \nabla \times (B \hat{k})$$

If the storm-relative wind vector, \vec{V} , points in the same direction as the vorticity vector, then we can write $\vec{\omega} = \lambda \vec{V}$ with λ a constant known as the abnormality. In this situation, use the vector identity below to show that vorticity advection is exactly balanced by vortex tilting and stretching. Comment on the physical significance of this result.

$$\nabla \times (\vec{V} \times \vec{\omega}) = \vec{V}(\nabla \cdot \vec{\omega}) - \vec{\omega}(\nabla \cdot \vec{V}) - (\vec{V} \cdot \nabla) \vec{\omega} + (\vec{\omega} \cdot \nabla) \vec{V}$$