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METR 4433 – Mesoscale Meteorology
Spring 2017

Problem Set #2
Linear Perturbation Theory and Mountain Waves

Distributed Tuesday, 31 January 2017
Due Thursday, 9 February 2017

INSTRUCTIONS: Please answer each of the questions shown below. Pay close attention to neatness and describe your work at each step of the solution process.

1. **Mountain Waves.** Determine the perturbation horizontal and vertical velocity for stationary gravity waves forced by flow over sinusoidally varying terrain given the following conditions

    - Height of the topography is $h(x) = h_m \cos(kx)$ where h_m = 50 m is a constant.
    - $B = 2.0\,x10^{-2}\,s^{-1}$
    - $u_o = 5\,m\,s^{-1}$
    - $k = 3\,x10^{-3}\,m^{-1}$

    Note that for small amplitude topography ($h_o k$<<1), the lower boundary condition can be approximated by $w' = \frac{Dh}{Dt} = u_o \frac{\partial h}{\partial x}\; at\; z = 0$.

2. **Wave Notation.** In the study of atmospheric wave motion, it is often necessary to consider the possibility waves that amplify or decay with time, as discussed in class. As noted in class, such waves have both a time-varying amplitude as well as a "wavy" part that describes the horizontal structure and motion. We can express this mathematically as

    $$u(x,t) = Ae^{\alpha t}\cos(kx - \omega t - kx_o)$$

    where A is the initial amplitude, α is the amplification factor (the inverse of which is the growth rate), k is the horizontal wavenumber, ω is the frequency, and x_o is the initial phase or location of the wave. Show that this expression can be written much more compactly as

    $$u(x,t) = \mathrm{Re}\{Be^{ik(x-ct)}\}$$

    where Re indicates the real part of the expression in {},both B and c are complex constants, and kc = ω. Determine the real and imaginary parts of B and c in terms of A, α, k, ω and x_o. In working this problem, you will see that the latter expression is simply a mathematical convenience for expressing the structure of a wave.

3. Wave Physics. In class, we discussed the so-called one-dimensional linear advection equation:

$$\frac{\partial u}{\partial t} + c\frac{\partial u}{\partial x} = 0 \ (c > 0 \, and \, const).$$

(a). Manipulate this equation to produce the classical one-dimensional wave equation, which has solutions corresponding to waves of arbitrary structure moving at speed c, one in the +x direction and one in the –x direction.

$$\frac{\partial^2 u}{\partial t^2} - c^2\frac{\partial^2 u}{\partial x^2} = 0 \ (c > 0 \, and \, const).$$

(b). Consider an initial profile of u given by u(x,t=0) = f(x), where f might be a sine or cosine, for example. If this profile is translated in the +x direction at speed c without changing shape (i.e., pure advection), then u = f(x-ct) is a solution to the above equation (recall we showed this in class, schematically, by riding on a wave at a constant phase location of x-ct and seeing what happened to x when t increases. Show that u = f(x-ct) indeed is a solution to the one-dimensional wave equation given above.
HINT: Let x' = x-ct and differentiate f using the chain rule.