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METR 4433 – Mesoscale Meteorology
Spring 2017

Problem Set #1
Linear Perturbation Theory and Mountain Waves

Distributed Tuesday, 17 January 2017
Due Tuesday, 31 January 2017

INSTRUCTIONS: Please answer each of the questions shown below. Pay close attention to neatness and describe your work at each step of the solution process.

1. **Linear Perturbation Theory.** Consider the governing equations shown below for 2D internal gravity waves:

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+w\frac{\partial u}{\partial z}+\frac{1}{\rho}\frac{\partial p}{\partial x}=0$$
$$\frac{\partial w}{\partial t}+u\frac{\partial w}{\partial x}+w\frac{\partial w}{\partial z}+\frac{1}{\rho}\frac{\partial p}{\partial z}+g=0$$
$$\frac{\partial u}{\partial x}+\frac{\partial w}{\partial z}=0$$
$$\frac{\partial \theta}{\partial t}+u\frac{\partial \theta}{\partial x}+w\frac{\partial \theta}{\partial z}=0$$

Assume the following basic state: u_o = constant, $\bar{\theta}(z)$, $\bar{p}(z)$, ρ_o = constant and $w_o = 0$, with the base state pressure and density in hydrostatic balance. Linearize the above equations about this basic state by assuming the following form for each of the dependent variables: $\xi(x,z,t)=\xi(basic\,state)+\xi'(perturbation)$. You should end up with the following **<u>(Hint: See Holton and Hakim's book)</u>**:

$$\frac{\partial u'}{\partial t}+u_o\frac{\partial u'}{\partial x}+\frac{1}{\rho_o}\frac{\partial p'}{\partial x}=0$$
$$\frac{\partial w'}{\partial t}+u_o\frac{\partial w'}{\partial x}+\frac{1}{\rho_o}\frac{\partial p'}{\partial z}-g\frac{\theta'}{\bar{\theta}}=0$$
$$\frac{\partial u'}{\partial x}+\frac{\partial w'}{\partial z}=0$$
$$\frac{\partial \theta'}{\partial t}+u_o\frac{\partial \theta'}{\partial x}+w'\frac{d\bar{\theta}}{dz}=0$$

2. **Linear Perturbation Theory**. Show how the four linearized equations derived in Problem #1 above can be combined to yield the following:

$$\left(\frac{\partial}{\partial t}+u_o\frac{\partial}{\partial x}\right)^2\left(\frac{\partial^2 w'}{\partial x^2}+\frac{\partial^2 w'}{\partial z^2}\right)+B^2\frac{\partial^2 w'}{\partial x^2}=0$$

where $B^2=\frac{g}{\overline{\theta}}\frac{\partial\overline{\theta}}{\partial z}$ with B known as the buoyancy frequency.

3. **Gravity Wave Frequency.** There exist two intrinsic frequencies in the atmosphere related to internal gravity waves. The first is the so-called buoyancy frequency, B, which you found in Problem #2 and is defined by

$$B^2=\frac{g}{\theta}\frac{\partial\theta}{\partial z}.$$

A very similar quantity, the Brunt-Vaisalla frequency, N, is defined by

$$N^2=-\frac{g}{\rho}\frac{\partial\rho}{\partial z}.$$

The two often are used interchangeably but in fact they are distinct and related mathematically as follows:

$$N^2=\left(\frac{g}{c_s}\right)^2+B^2$$

where c_s is the speed of sound given by $c_s=\sqrt{\gamma RT}=\sqrt{\frac{\gamma p}{\rho}}$, and where all symbols have their conventional meaning.

Verify the relationship between B^2 and N^2 by showing that the right and left hand sides are equal, and comment why, in an incompressible atmosphere, the two are identical.