

## Lake Effect Snow Storms

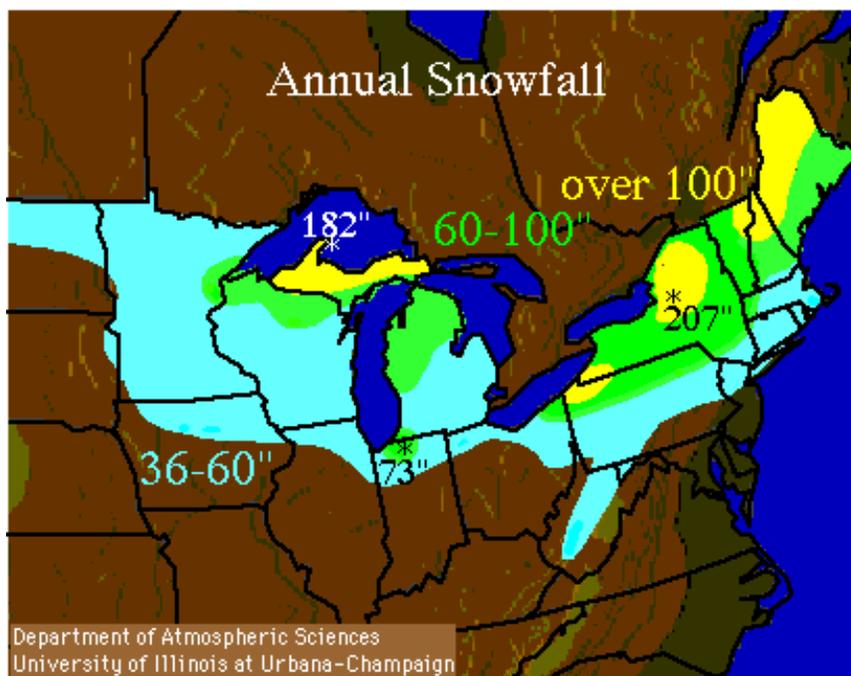
### METR 4433, Mesoscale Meteorology

### Spring 2006

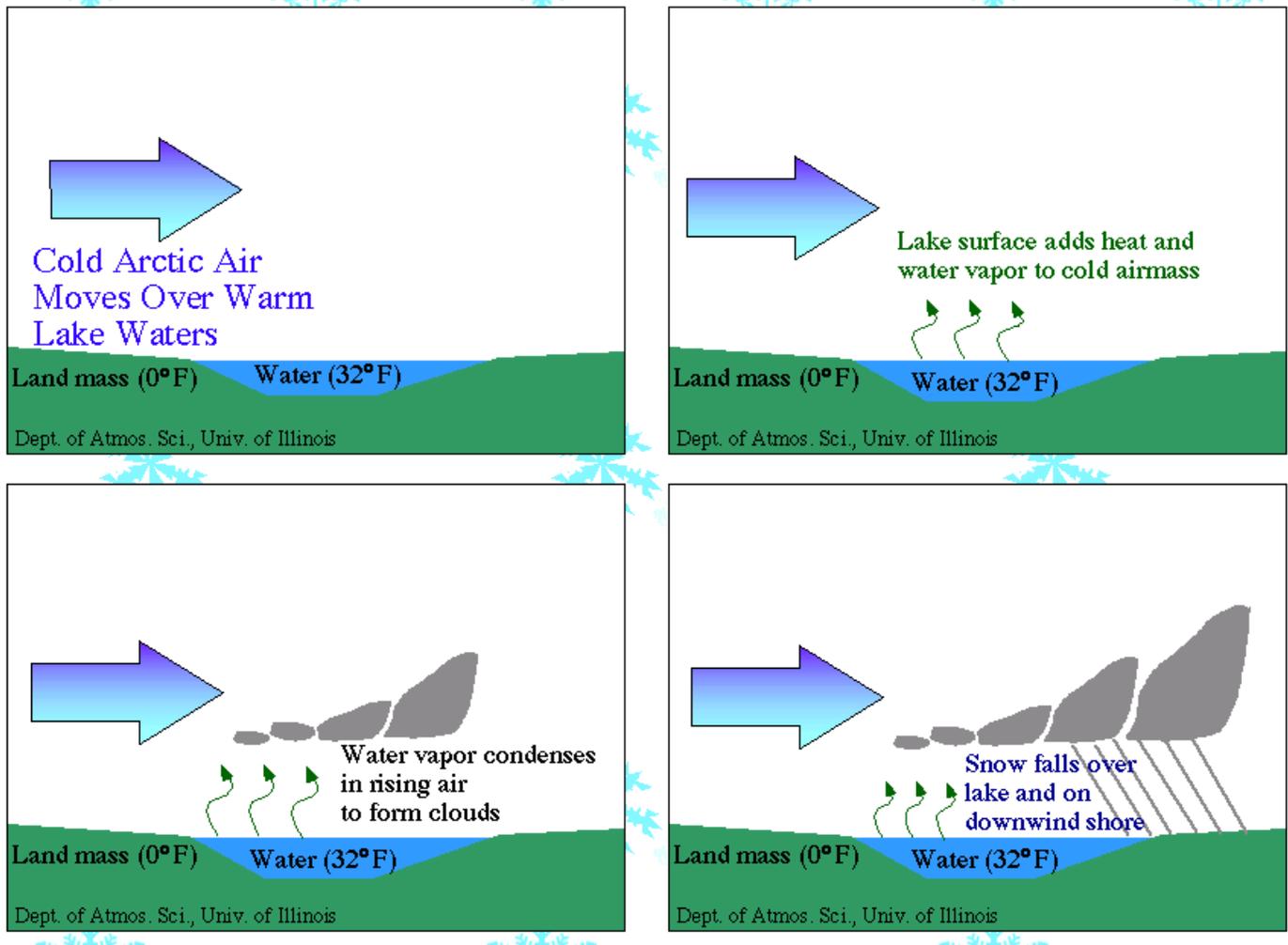
(some of the material in this section came from Bart Geerts)

**Definition:** The term “lake effect” generally refers to the effect of any lake in modifying the weather about its shore and for some distance downwind. In the United States, this term is applied specifically to the region about the Great Lakes or the Great Salt Lake. More specifically, lake effect often refers to the generation of sometimes spectacular snowfall amounts to the lee of the Great Lakes as cold air passes over the lake surface, extracting thermodynamic energy and moisture, resulting in cloud formation and snowfall downwind of the lake shore.

For instance, 50 cm of snow may accumulate over the course of a few days near the shore, and 50 km from the lake shore the ground may be bare. Lake-effect snow occurs elsewhere as well, e.g. near Lake Baikal in Russia, but nowhere is it so pronounced and has it such an effect on ground and air transportation. The local maxima in snowfall are not due to the proximity of mountains or an ocean. The difference is not because the southern and eastern shores are cooler than the surroundings; in fact they are slightly warmer than the other shores. Snowfall typically occurs in this area after the passage of a cold front, when synoptic factors are not conducive to precipitation.



**Overview of Lake Effect Process:** Shown below is a schematic diagram of how lake-effect snowfall is generated. Note that the temperatures shown are arbitrary. Lake-effect snow actually is more effective if both land and water temperatures are higher.



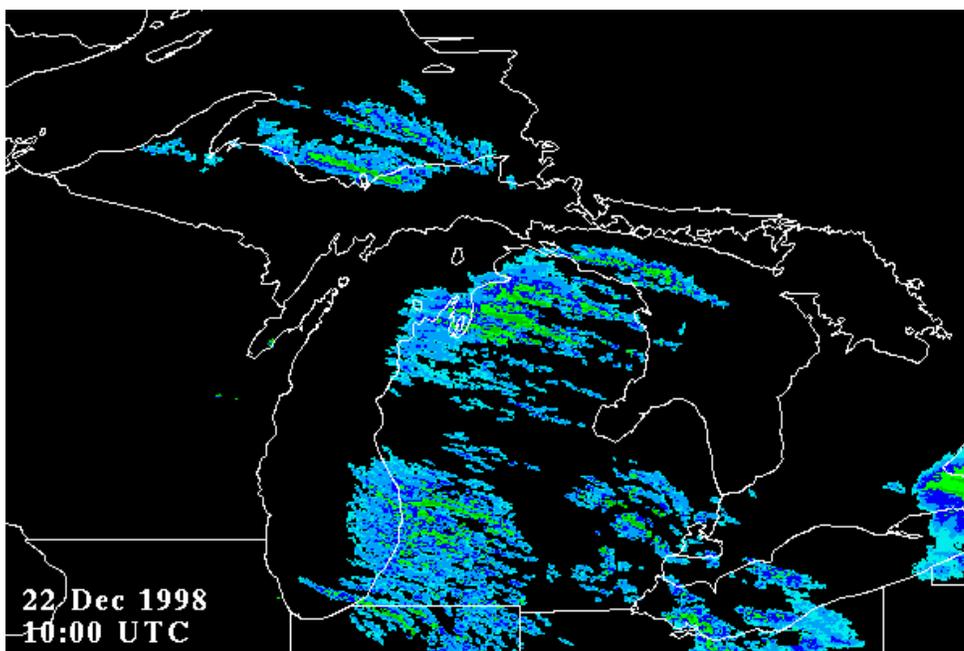
In more detail, these are the mechanisms, ranked in usual order of importance

- **Heating.** The water of the Great Lakes lags behind the atmosphere in cooling through the fall and early winter. The heating from below results in static instability, especially during cold outbreaks. This instability mixes near-surface warm, moist air into the lowest 1 to 1.5 km, sometimes more. Rising air quickly reaches saturation, and the result is shallow cumuliform clouds, often aligned in bands parallel to the low-level wind. By January, ice covers most lakes, at least in part, cutting off or reducing the heat supply. Lake Erie often freezes entirely because it is more shallow.
- **Moisture.** The lake surface evaporates, which is very effective when the wind is strong and the air dry (Dalton's equation, Note 4.E). The cold air from Canada has

a very low vapor pressure. Also, also strong winds cause spray, facilitating evaporation.

- **Wind Fetch.** The length of trajectory of the wind across the lakes has a great bearing on the development of lake-effect snow. The greater distance the wind blows over the warm water, the greater the convection. Three of the five lakes, those with the most population centers, are relatively long and narrow. Winds blowing along the length of these lakes have a long trajectory over water, whereas a 30 degree windshift will bring the winds across the lake, not only shortening the trajectory considerably, but also moving the lake-effect snow to a different site
- **Frictional Difference.** The stress applied to the atmosphere from the surface is much greater over a rough land surface than over a relatively smooth lake water surface. When the surface winds blows from lake to land, it encounters increased friction, slowing the surface wind over the land, resulting in surface convergence and lifting. Since stress varies with the square of the wind speed, this effect is greater with strong winds.
- **Upslope lift.** In some localities, wind blowing from a lake onshore is forced to climb up hills. This is not a major factor in precipitation along the immediate lakeshore, but affects some more island locations. Certainly this effect is important in the case of Lake Baikal in Siberia.
- **Land breeze.** Sometimes the lake-effect snow is concentrated along a narrow band due to mesoscale flows around the lake, in particular a landbreeze from one or opposing shores, e.g. when a weak northerly gradient wind blows along Lake Michigan.
- **Large-scale forcing** (potential vorticity advection, isentropic uplift ...). The general cyclonic nature of an airmass, which supports development of precipitation anywhere, may enhance lake-effect snow.

**Dynamics of Mesoscale Bands (From Emanuel, 1994).** Lake effect snow storms, and in fact many snow storms, tend to be organized in parallel bands – giving the often observed temporal intermittence that is characteristic with such storms.



The dynamics of mesoscale bands can be explained by symmetric instability, which is related to slantwise convection. The most common type of convection is due to an unstable distribution of mass in the vertical (along the direction of gravitational acceleration), i.e., warm, moist air underneath cold, dry air. The behavior of a fluid in this state is governed by the theory of static stability (i.e., parcel theory), and in it potential temperature (of various types) is conserved.

In addition to gravitational acceleration on a rotating planet, however, there exists centrifugal (inertial) acceleration. Centrifugal instability is nothing more than static instability with the centrifugal acceleration taking the place of gravity. In centrifugal instability, the conserved variable is angular momentum rather than potential temperature. Slantwise convection is the mechanism by which symmetric instability is released.

To understand symmetric instability, consider the equations of motion with a constant Coriolis parameter (i.e., on an f-plane)

$$\begin{aligned}\frac{Du}{Dt} &= -\frac{1}{\rho} \frac{\partial p}{\partial x} + fv \\ \frac{Dv}{Dt} &= -\frac{1}{\rho} \frac{\partial p}{\partial y} - fu \\ \frac{Dw}{Dt} &= -\frac{1}{\rho} \frac{\partial p}{\partial z} - g\end{aligned}$$

Let's now assume that the pressure gradient force in the y-direction is a constant, independent of height. Thus, the geostrophic wind,  $u_g$ , is constant well, i.e.,

$$fu_g = -\frac{1}{\rho} \frac{\partial p}{\partial y} = \text{const}. \text{ The equations therefore become}$$

$$\begin{aligned}\frac{Du}{Dt} &= -\frac{1}{\rho} \frac{\partial p}{\partial x} + fv \\ \frac{Dv}{Dt} &= f(u_g - u) \\ \frac{Dw}{Dt} &= -\frac{1}{\rho} \frac{\partial p}{\partial z} - g\end{aligned}$$

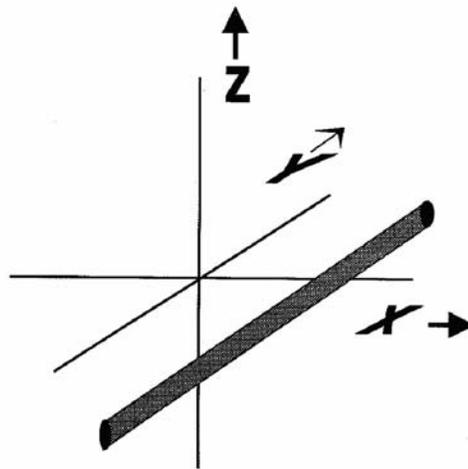
If our coordinate system is now allowed to move with speed  $u_g$ , we have

$$\begin{aligned}\frac{Du}{Dt} &= -\frac{1}{\rho} \frac{\partial p}{\partial x} + fv \\ \frac{Dv}{Dt} &= -fu \\ \frac{Dw}{Dt} &= -\frac{1}{\rho} \frac{\partial p}{\partial z} - g\end{aligned}$$

By definition,  $u = dx/dt$ , and thus the second equation in the last set above can be written  $\frac{Dv}{Dt} = -f \frac{dx}{dt}$ , or equivalently,  $\frac{DM}{Dt} = 0$  where  $M \equiv v + fx$ . The quantity  $M$  is called the **absolute momentum** and it is conserved for inviscid flow on an  $f$ -plane when one component of the geostrophic wind is constant (here the zonal component). In this case, only the meridional component of the geostrophic wind can vary with height, which means the temperature gradient is orthogonal to it via the thermal wind relation. Consequently, the  $y$ -direction lies along isotherms on a surface of constant pressure and the  $x$ -direction is oriented along the temperature gradient, pointing toward *warmer* air.

Considering the environment to be geostrophic, suppose we now perturb it by displacing a two-dimensional “tube” of air in the  $x$ -direction (see figure below). In order for  $M$  of the tube (analogous to the potential temperature of a parcel in parcel theory) to be conserved, the geostrophic wind in the  $x$ -direction ( $u_g$ ) must remain unperturbed by the displacement. If  $p'$  is the perturbation pressure associated with the displacement and  $u_g'$  is the associated geostrophic wind perturbation, then the equation shown above, applied to the perturbation, is

$$fu'_g = -\frac{1}{\rho} \frac{\partial p'}{\partial y} = 0.$$



The only disturbances that satisfy this relation are those which have no meridional structure, i.e., are elongated in the  $y$ -direction. We thus can write the zonal momentum equation as

$$\frac{Du}{Dt} = -\frac{1}{\rho} \frac{\partial p}{\partial x} + fv = f(v - v_g) = f(M - M_g)$$

where  $M_g \equiv v_g + fx$ . This quantity is called the **geostrophic absolute momentum** and you can think of it as the absolute momentum of the environment. If the displaced tube, as in parcel theory, does not disturb the environment, then the distribution of  $M_g$  remains

fixed. We see that the flow is unstable (accelerates in the x-direction) if  $M - M_g > 0$ . In other words, if the tube is given a small displacement in the x-direction, it will accelerate in that direction if the value of  $M$  in the tube is larger than  $M$  of the fluid into which it is displaced ( $M_g$ ).

Because  $M$  is conserved, the zonal acceleration acts in a direction opposite to the displacement when  $M_g$  increases in the x-direction. We can formalize this via a Taylor series for small displacements:

$$M_g(x + \delta x) = M_g(x) + \frac{\partial M_g}{\partial x} dx.$$

Let's now assume that  $M(x + \delta x) = M_g(x)$ , i.e.,

$$M(x + \delta x) = M_g(x) = M_g(x + \delta x) - \frac{\partial M_g}{\partial x} dx$$

Using this in the last equation on the previous page gives

$$\frac{Du}{Dt}_{x+\delta x} = -f \frac{\partial M_g}{\partial x} dx$$

Thus, if  $M_g$  decreases in the x-direction, the flow will be unstable. This is a special case of centrifugal instability of a straight geostrophic flow on an f-plane and is exactly analogous to parcel theory. In parcel theory, the conservation of potential temperature means that the vertical pressure gradient force and gravity will not generally be in balance when a parcel is displaced vertically. If the resulting acceleration is in the direction of the displacement, the flow is unstable. In centrifugal convection, conservation of some angular momentum-like variable (here,  $M$ ) means that when a ring of fluid is displaced in a direction orthogonal to the axis of rotation (here  $z$ ), the pressure gradient and centrifugal (or Coriolis) acceleration generally will not be in balance. If the net acceleration is in the direction of the displacement, then the flow is unstable to centrifugal convection.

Let's now take the tube of air shown in the preceding figure and displace it *vertically* as well as horizontally, again assuming that it does not disturb the pressure of the environment. The equations governing the acceleration of the tube are (see your thermodynamics notes for the vertical equation of motion)

$$\frac{Du}{Dt} = f(M_{tube} - M_g)$$

$$\frac{Dw}{Dt} = g \left( \frac{\alpha_{tube} - \alpha_{environ}}{\alpha_{environ}} \right)$$

We can rewrite the RHS of the vertical equation in terms of entropy and lapse rate to yield

$$\frac{Du}{Dt} = f(M_{tube} - M_g)$$

$$\frac{Dw}{Dt} = \Gamma(s_{tube} - s_{environ})$$

where  $\Gamma$  = lapse rate (saturated and unsaturated, as appropriate) and  $s$  is the entropy =  $c_p \ln \theta$ , where the potential temperature is either the standard potential temperature for unsaturated air or the equivalent potential temperature for saturated air. As is true for parcel theory, the acceleration is proportional to the difference between a quantity that is conserved in the displacement of the tube and the same quantity in the tube's environment. This clearly shows the dynamical equivalence of the centrifugal and gravitational acceleration. Keep in mind that this analysis is valid only for a tube of air elongated in the direction of the thermal wind (here, the y-direction). Also, remember that  $s_{tube}$  is always conserved following the displacement of the tube.

**Application.** Consider a typical atmospheric situation in which the temperature increases in the x-direction in the Northern Hemisphere. Associated with this is a meridional flow that increases in altitude (that is,  $v_g$  and thus  $M_g$  both increase with height). Let's now suppose that the flow is stable to both gravitational and centrifugal instabilities. This means that  $M_g$  must increase in the x-direction or

$$\frac{\partial M_g}{\partial x} = f + \frac{\partial v_g}{\partial x} > 0.$$

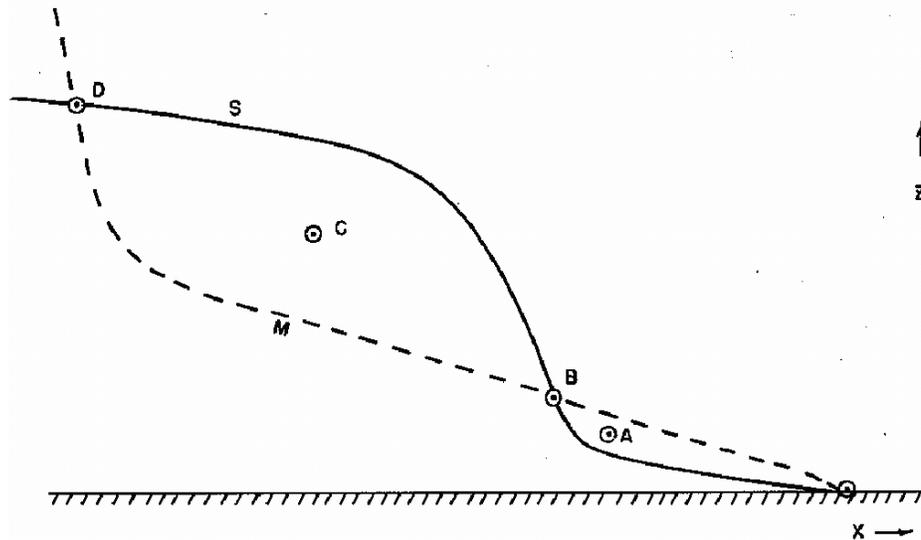
In other words, the geostrophic absolute vorticity (remember that the zonal geostrophic wind is constant) must have the same sign as  $f$ . Under these conditions, we show below that the flow can be **unstable** to slantwise convection even though it is stable to purely lateral (centrifugal) as well as vertical (gravitational) parcel displacements.

Consider the figure on the next page, which shows hypothetical distributions of  $M$  (dashed) and  $s$  (solid) for the environment (i.e., think of them as  $M_g$  and  $s_{environ}$ , just like curves on a thermodynamic diagram). Keep in mind that the atmosphere is statically stable, and thus  $s_{environ}$  increases with height. Analogous to parcel theory, let's examine the stability of the atmosphere to a finite displacement of a particular "sample" of air (here a tube rather than a parcel) – indicated by the circle containing a dot at the ground in the lower right part of the figure. Remember that you can think of  $s$  as potential temperature.

Owing to the horizontal temperature gradient of the environment, however,  $s_{environ}$  increases in the x-direction and  $M_g$  increases in the vertical because  $v_g$  does as well. Thus, both  $M_g$  and  $s_{environ}$  increase upward and in the positive x-direction and thus surfaces of constant  $M_g$  and  $s_{environ}$  slope upward in the negative x-direction.

If the tube is displaced vertically upward, then a downward acceleration will occur because  $s_{tube}$ , which is constant (just like the assumption that an unsaturated parcel moves

dry adiabatically), is moving into a larger value of  $s_{\text{environ}}$ . This creates a negative buoyancy force and thus a downward acceleration.



**Fig. 12.3** Hypothetical distributions of  $M$  (dashed line) and neutral-buoyancy (solid line) surfaces for a tube located at the lower right. Domain is approximately  $100 \times 10$  km.

Consider the  $M_g$  and  $s_{\text{environ}}$  lines that pass through the surface parcel shown in the figure. This particular  $M_g$  surface is one along which the displaced tube experiences no horizontal acceleration since its value of  $M$ , which is conserved in the displacement, is always the same as the environment. This is analogous to a parcel rising adiabatically in a neutral environment with the same potential temperature as the environment.

Similarly, the particular  $s$  surface shown is one along which the tube experiences no vertical acceleration. Thus, it is evident that the tube, when displaced an arbitrary distance in some direction, will accelerate vertically toward its  $s$  surface and horizontally toward its  $M$  surface.

By this simple rule, if the sample tube is displaced to position A in the figure, it will experience a downward and rightward acceleration which acts to return it to its initial location. If displaced to B, the tube experiences no acceleration. When displaced beyond B, say to C, it is subject to an upward and leftward acceleration *away* from its initial location. Finally, at D, the tube experiences no acceleration (same as starting point at the ground).

The state of the atmosphere in the figure is therefore said to be conditionally unstable to slantwise convection – the condition being the displacement of the tube beyond point B – the point of free convection. This is exactly analogous to standard vertical parcel theory but is now extended to two dimensions and under the assumption of a tube of air rather than a “point” parcel.