Dr. Droegemeier

METR 4433 – Mesoscale Meteorology
Spring 2017

Key to Examination #2

Given Thursday, 30 March 2017

On my honor, I affirm that I have neither given nor received inappropriate aid in completing this examination.

NAME (please print): ____

Signature: ____ Date: _________________

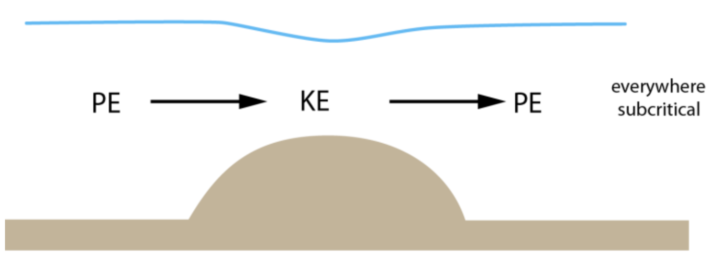
Please read each question carefully before answering and be sure to complete all parts of multi-part questions. You have 75 minutes to complete the exam, which is worth 200 points. Please show all of your work in the space provided.

1. (40 points) *Consider the situation of SUBCRITICAL flow in the image below, which represents a terrain feature. Using the equation provided, illustrate (on the diagram) AND explain what happens to the (a) depth of the fluid, (b) speed of the fluid, and (c) the energy (kinetic and potential) as the flow passes over the terrain feature from left to right. More space is provided on the next page.*

$$\left(1-Fr^2\right)\frac{\partial D}{\partial x} = -\frac{\partial h_t}{\partial x}, \text{ where } Fr^2 = \frac{u^2}{c^2}$$

with c being the shallow water gravity wave phase speed.

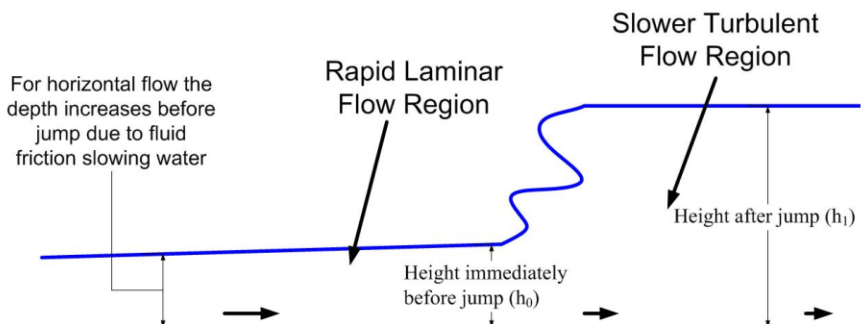
As flow approaches the terrain feature from west to east (left to right), it encounters a positive terrain slope, which means the RHS of the above equation is negative. Given that the flow is sub-critical (Fr < 1), the term in parentheses on the LHS of the equation is positive. Consequently, $\partial D / \partial x < 0$ and the depth of the flow decreases toward the peak of the feature with kinetic energy increasing. At the top of the terrain feature, the horizontal wind reaches its peak amplitude and the fluid its minimum depth, thus with maximum kinetic energy and minimum potential energy. To the right of the center of the terrain feature, the terrain slope changes sign and thus the flow once again thickens downstream, with potential energy increasing at the expense of kinetic energy.



2. (20 points) *What is a hydraulic jump and what role does it play in downslope wind storms?*

A hydraulic jump is a phenomenon in which rapidly-moving fluid, such as water, enters a region in which the gravity wave speed is less than the speed of the oncoming fluid. In this situation, the fluid "piles up," creating a region of turbulence that dissipates kinetic energy and an associated increase in fluid depth. The role it plays is to slow the flow down and dissipate energy.

(Not needed) Hydraulic jumps are sometimes induced by placing a barrier or obstacle in the flow, such as in a drainage channel, to slow the speed of water in the event of rapid flooding.



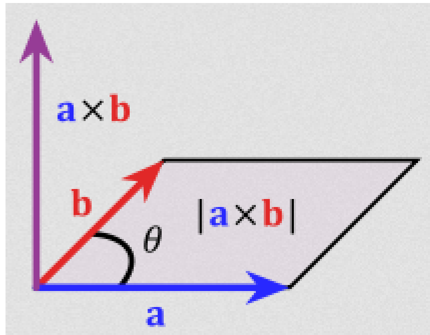
3. (40 points) *Consider the three scenarios below, which show pressure (solid lines, millibars) and density (dashed lines, kg m^{-3}) contours in a vertical cross-section. Making use of the circulation equation also provided, answer the following questions in the context of a sea breeze: (a) In which scenario is the change in circulation strongest, and why? (b) What is the direction of the change in circulation in that case, and why? Space is provided on the next page.*

$$\frac{dC}{dt} = -\iint \left[\nabla\left(\frac{1}{\rho}\right) \times \nabla p \right] \bullet \hat{n}\, dA$$

where $\hat{n}$ is the outward-directed unit normal vector to the area A.

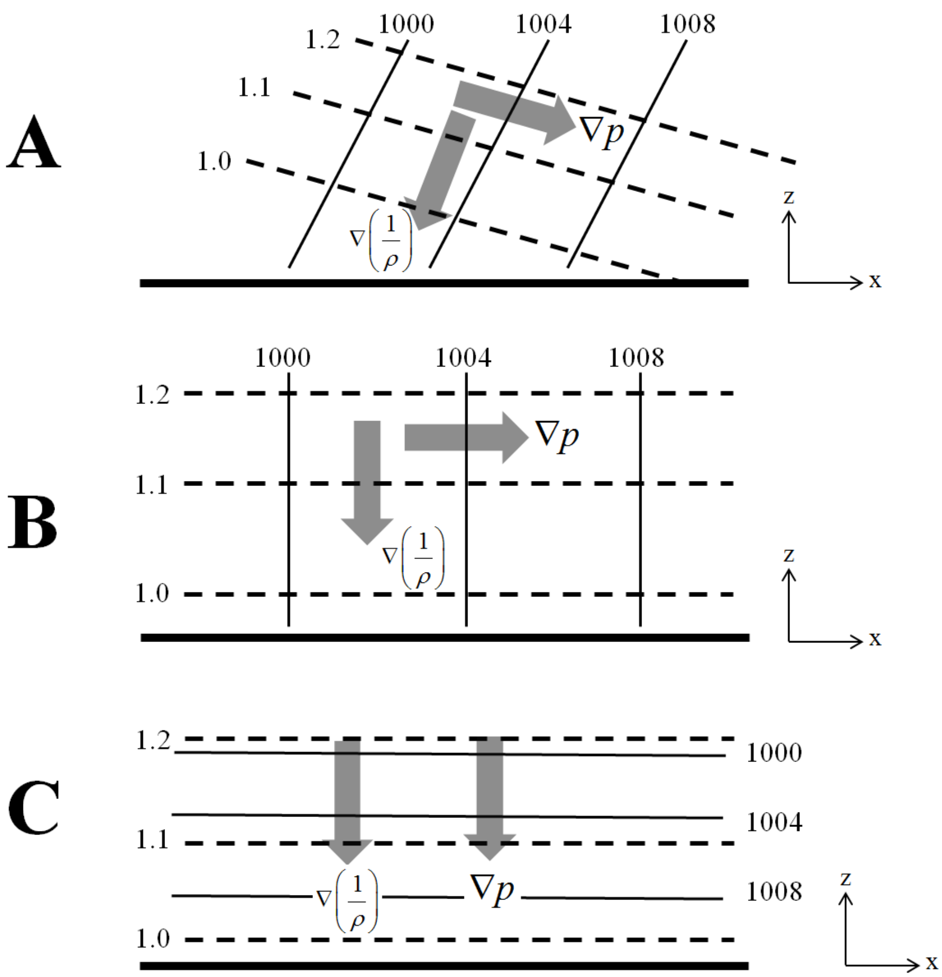
Drawn on the figure below are the gradients involved in the circulation equation. Note that the gradient of the inverse of the density points in a direction opposite to the direction of the gradient.

(a) The maximum change in circulation is the one in which the cross product is maximum, that is, the area of the trapezoid created by the contours is the largest (recall that the cross product of two vectors, A and B, is the area traced out by the trapezoid of the two vectors – see figure below). Also, the sine of the angle between the two vectors is a maximum. This occurs in case B, where the angle is 90 degrees and the areas in B are clearly larger than the areas in A.



(b) The change in circulation in this situation is in the clockwise direction, with the thumb pointing into the page by the right hand rule.

Parenthetically (not required in the answer), the circulation change in case C is zero because the two gradient vectors are parallel.



***4. (40 points)** Using illustrations, explain the diurnal cycle of the southern Plains low-level jet. The following equation might be useful.*

$$\frac{\partial \vec{V}_g}{\partial p} = -\frac{R}{fp} k \times \nabla_p T$$

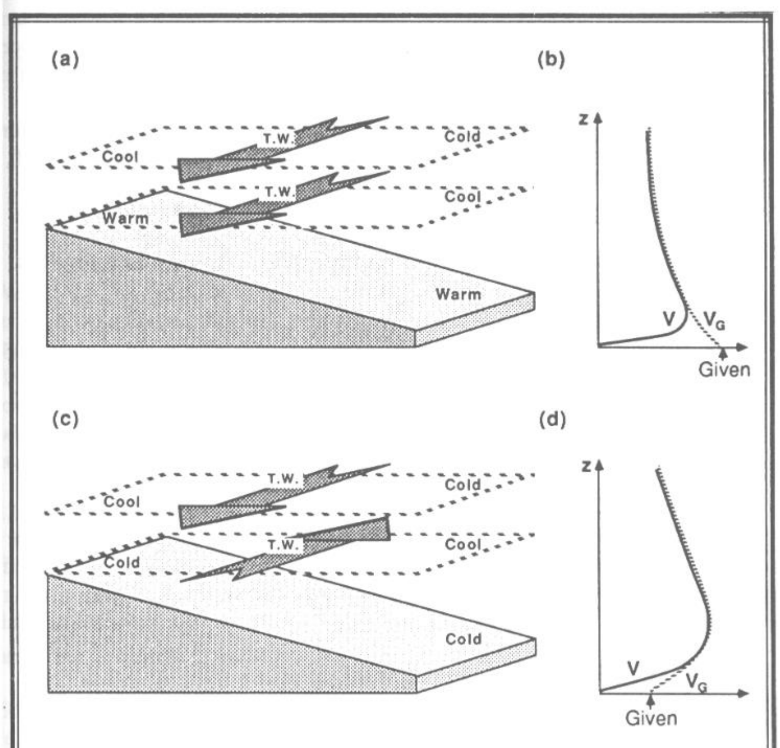


Fig. 12.17 Diurnal oscillation of the low level jet due to thermal wind (T.W.) forcings over sloping terrain.

Daytime (a, b):

Solar insolation warms ground and forms mixed layer with near-adiabatic lapse rate so that west-to-east temperature gradient – measured along a horizontal surface – is negative $\frac{\partial T}{\partial x} < 0$ near the ground and aloft. So thermal wind: $\frac{\partial v_g}{\partial z} < 0$. Mixing is strong during the daytime and tends to blur the effect so that you don't get a jet.

Nighttime (c, d)

Ground cools more quickly than the air and the gradient is reversed, west-to-east temperature gradient – measured along a horizontal surface – is positive $\frac{\partial T}{\partial x} > 0$ near the ground. So thermal wind: $\frac{\partial v_g}{\partial z} > 0$.

Above the level of the inversion the gradient may again reverse such that $\frac{\partial v_g}{\partial z} < 0$.

Near the ground frictional forces decrease the wind speed. This leads to the formation of a jet at the level of nocturnal inversion. The stability of the air (lack of mixing) below the inversion helps the jet to persist.

5. (20 points) *Briefly describe the differences among the following terms in the buoyancy force of the vertical momentum equation: Thermal buoyancy, vapor buoyancy, and water loading. The equation below may be useful.*

$$\frac{dw}{dt} = -\frac{1}{\bar{\rho}}\frac{\partial p'}{\partial z} + g\left(\frac{\theta'}{\bar{\theta}} + 0.61q_v{}' - q_L\right)$$

Thermal buoyancy – is the force per unit mass resulting from a temperature difference between a parcel and its environment. When this difference is positive, i.e., when the parcel is warmer than its environment, the buoyancy force acts to accelerate the updraft (dw/dt > 0).

Vapor buoyancy – is the force per unit mass resulting from a difference in moisture content, measured by the water vapor mixing ratio, between an air parcel and its environment. If the parcel is moister than its environment (qv' > 0), the vapor buoyancy force contributes to updraft acceleration (dw/dt > 0). This is consistent with the fact that a moist parcel is less dense, or more buoyant, than its environment at the same temperature.

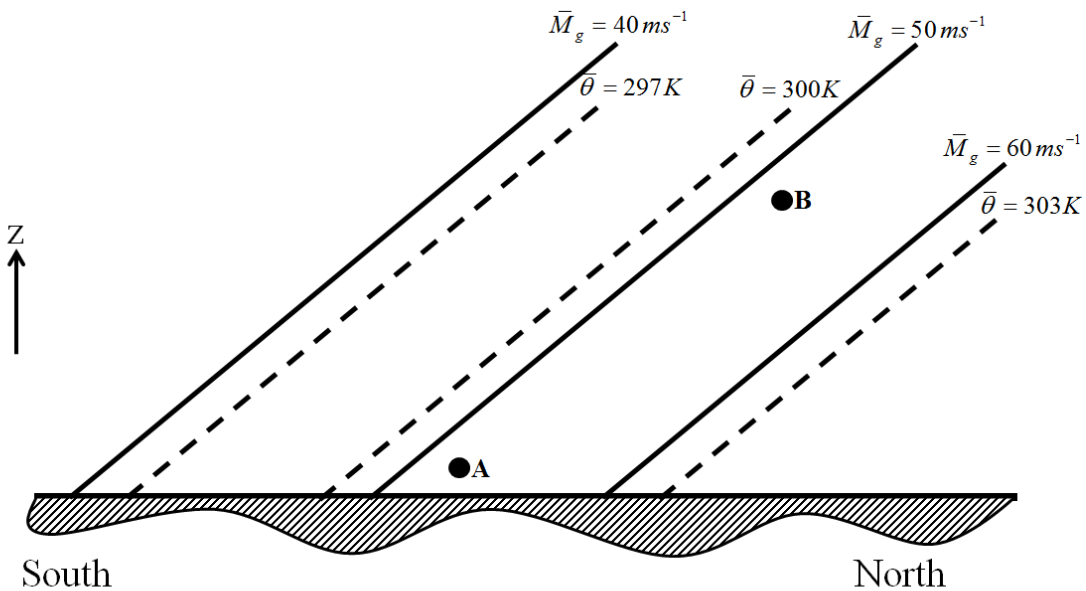
Water loading – is the force per unit mass resulting from the presence of liquid water (qL) in an air parcel. Because qL, the liquid water mixing ratio, is always positive, this contribution is negative definite and always acts to decelerate an updraft (dw/dt < 0).

6. (40 points) *For the vertical cross section below, which shows environmental momentum (solid) and potential temperature (dashed) contours, assess and explain the instability condition of a tube of air, oriented in and out of the page, to VERTICAL, HORIZONTAL, and LATERAL (from point A to B) displacements. The equations below may be helpful, and more space is provided on the next page.*

$$\frac{d^2(\Delta z)}{dt^2} = \frac{g}{T_o}(\gamma - \Gamma_d)\Delta z \qquad \frac{d^2(\Delta r)}{dt^2} = -\frac{1}{r_o^3}\frac{d\bar{M}^2}{dr}\Delta r \qquad \frac{d^2(\Delta y)}{dt^2} = -f\left(f - \frac{\partial u_g}{\partial y}\right)\Delta y$$

$$\frac{d^2(\Delta s)}{dt^2} = \left\{ f\frac{\partial \bar{M}_g}{\partial z}\left[\left(\frac{\Delta z}{\Delta y}\right)_{tube} - \left(\frac{\Delta z}{\Delta y}\right)_{\bar{M}_g}\right]\cos(\alpha) + B^2\left[\left(\frac{\Delta z}{\Delta y}\right)_{\bar{\theta}} - \left(\frac{\Delta z}{\Delta y}\right)_{tube}\right]\sin(\alpha)\right\}\Delta s\cos(\alpha)$$

Referencing the diagram on the next page, a tube of air displaced horizontally left to right will be inertially unstable because it is carrying with it a value of absolute angular momentum smaller than that of the environment into which it is moving. The tube therefore will accelerate away from its original location. A tube of air displaced vertically upward will be unstable because it is carrying with it a potential temperature that is larger in value than that of the environment into which it is moving. A tube of air displaced laterally from A to B will be inertially neutral because it is carrying with it the same values of potential temperature and absolute angular momentum as those of the environment into which it is moving. In this case, the slopes are all identical in the last equation above, leading to zero acceleration.



$\bar{M}_g = 40\,ms^{-1}$
$\bar{\theta} = 297K$
$\bar{M}_g = 50\,ms^{-1}$
$\bar{\theta} = 300K$
$\bar{M}_g = 60\,ms^{-1}$
$\bar{\theta} = 303K$
B
Z
A
South
North