Dr. Droegemeier

METR 4433 – Mesoscale Meteorology
Spring 2017

SOLIUTION KEY to Examination #1

Thursday, 16 February 2017

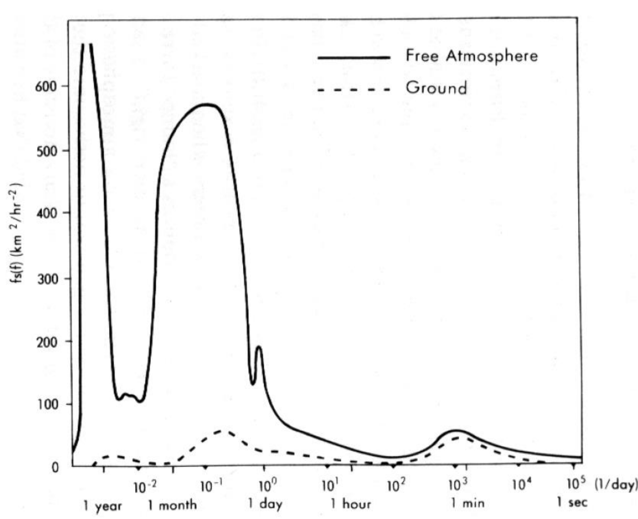
On my honor, I affirm that I have neither given nor received inappropriate aid in completing this examination.

Signature: ____ Date: __________________

Please read each question <u>carefully</u> before answering and be sure to complete all parts of multi-part questions. You have 75 minutes to complete the exam, which is worth 200 points. Please show all of your work in the space provided.

1. (25 points) Characterize the term "mesoscale" in light of the general amount of kinetic energy present in, and transferred among, atmospheric scales ranging from large (planetary) to small (turbulence). What is the role of the mesoscale? Use illustrations and analogies as needed.

The mesoscale is associated with a portion of the energy spectrum characterized principally by the transfer of energy from larger (e.g., synoptic) to smaller (e.g., storm) scales. That is, relatively little energy actually resides in the mesoscale, but instead passes through. A good analogy is a large tank of water (representing large energy on large spatial scales, such as synoptic and above) emptying, vio a pipe, into a smaller tank (representing smaller energy on smaller spatial scales, such as storm and below). The pipe, through which the water flows but does not spend much time, represents the mesoscale.



2. (25 points) Why can a gravity wave, in the mesoscale, be considered hydrostatic whereas a severe thunderstorm, in the storm scale, cannot? Use equations if you wish.

Very simply, features on the mesoscale, such as internal or external gravity waves, do not typically exhibit large vertical accelerations whereas updrafts in severe thunderstorms do. Even hurricanes, which contain thunderstorms that are non-hydrostatic, can be considered hydrostatic in terms of their overall dynamics.

In equation form, the hydrostatic equation emerges from the vertical equation of motion:

$$\frac{dw}{dt} = -\frac{1}{\rho}\frac{\partial p}{\partial z} - g$$

When the vertical acceleration, dw/dt, is small or zero, the hydrostatic equation is what remains:

$$-\frac{1}{\rho}\frac{\partial p}{\partial z} = g$$

One can perform a scale analysis (NOT required for the solution here) to show that mesoscale features are hydrostatic and storm-scale features are non-hydrostatic.

On the mesoscale, we have the following scales: H ~ 10 km, w ~ 100 cm/s (0.1 m/s), T ~ 10,000 sec, $g = 10\ m/s^2$, $\rho = 1\ kg/m^3$, and Δp = 1000 mb = 100,000 Pa. Using them in the above equation, we find that

$$\frac{dw}{dt} = -\frac{1}{\rho}\frac{\partial p}{\partial z} - g$$

$$\frac{0.1}{10^4} = \frac{1}{1}\frac{10^5}{10^4} - 10$$

Clearly, the acceleration term, dw/dt, is much smaller than the other two terms and thus, on the mesoscale, motions are nearly hydrostatic.

On the storm-scale, all of the parameters are the same, with the exception of the following: w ~ 10 m/s, T ~ 1,000 sec. Also, we use the Boussinesq form to get a buoyancy term. Using these in the above equation, we find that

$$\frac{dw}{dt} = -\frac{1}{\rho_o}\frac{\partial p'}{\partial z} + g\frac{\theta'}{\overline{\theta}}$$

$$\frac{10}{10^3} = \frac{1}{1}\frac{10^2}{10^4} - 10\frac{1}{300}$$

$$10^{-2} \qquad 10^{-2} \qquad 3x10^{-2}$$

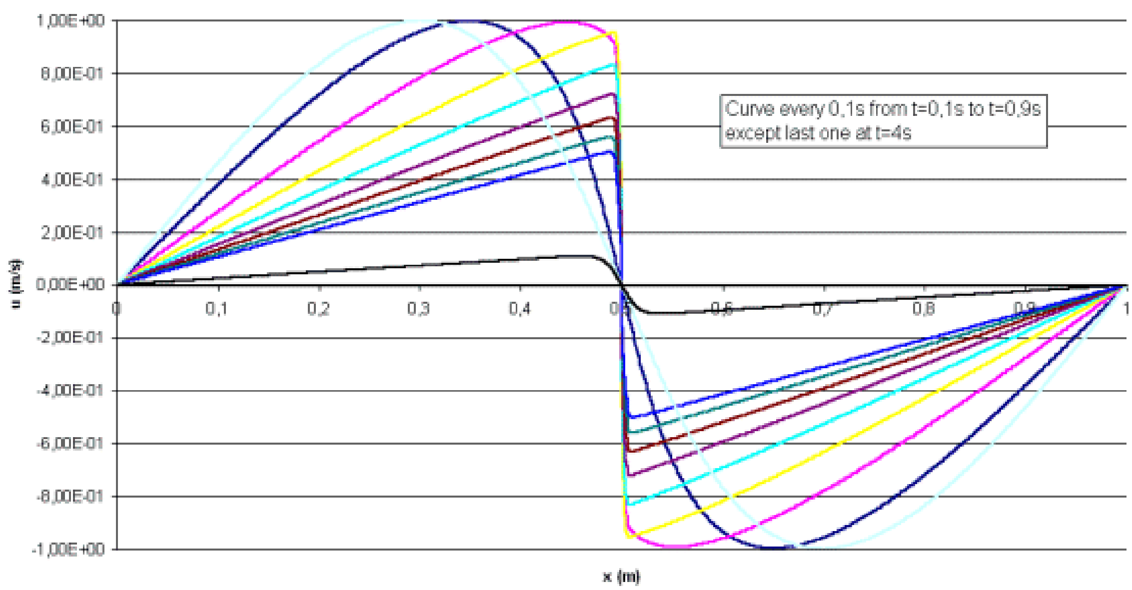
Clearly, the acceleration term, dw/dt, cannot be neglected on the storm scale.

3. (30 points) (a) Describe the principal difference(s) between a linear and a nonlinear system or phenomena and give one example of each. The examples need not be meteorological.

(a). In a nonlinear system, feedbacks occur – everything is connected to everything else. In a linear system, everything is isolated – one thing does not affect another thing. An example of a linear system would be a pure advection equation in which the advection speed is a constant, and thus a sine wave simply moves uniformly without changing shape, or a cloud that does not affect its surroundings. A nonlinear system would be advection in which the quantity being advected is the same as the quantity doing the advecting, e.g., horizontal wind. In this case, a sine wave, for example, steepens with time. Another example is a drug addict, who takes drugs, which creates a greater craving for them, which leads to them taking more drugs, which enhances the dependence, etc. Thus, a feedback occurs, i.e., the need is linked to the amount taken, and the amount taken responds to the need.

(b) What is non-linear steepening and what significance does it play in the atmosphere?

It occurs owing to advection of the wind by itself, which means different parts of a wave move at different speeds (can show drawing). As a result, the wave does not move uniformly but tends to steepen with time. In the atmosphere, such steepening is important for the creation of strong gradients, such as at gust fronts or in larger scale fronts. But the gradient can't become infinitely steep because as steepening continues, turbulence sets in to reduce the gradient.



4. (60 points) Consider the following system of equations:

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}=-\frac{1}{\rho}\frac{\partial p}{\partial x}+fv$$
$$\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}+w\frac{\partial v}{\partial z}=-\frac{1}{\rho}\frac{\partial p}{\partial y}-fu$$
$$\frac{\partial \theta}{\partial t}+u\frac{\partial \theta}{\partial x}+v\frac{\partial \theta}{\partial y}+w\frac{\partial \theta}{\partial z}=0$$

Linearize this system around a basic state of horizontal wind that is only a function of height ($\overline{u}(z)\, and\, \overline{v}(z)$), potential temperature and pressure that are functions of height ($\overline{P}(z)\, and\, \overline{\theta}(z)$), and density that is constant ($\rho_o = const$). You do NOT need to solve for the frequency or combine the equations - simply linearize them. Additional space is available on the next page.

We assume the following form for the dependent variables. Note that I did not specify a base state for the vertical velocity, but it's always equal to zero. Some students may have made it non-zero, which is fine, but if they did, we can explain in class that doing so does not make physical sense. They don't necessarily need to write down the next step, where each variable is listed below, but they definitely need to show their work in substituting these variables into the governing equations.

$$u(x,y,z,t)=\overline{u}(z)+u'(x,y,z,t)$$
$$v(x,y,z,t)=\overline{v}(z)+v'(x,y,z,t)$$
$$w(x,y,z,t)=\qquad w(x,y,z,t)$$
$$\theta(x,y,z,t)=\overline{\theta}(z)+\theta'(x,y,z,t)$$

Note that we don't have to worry about the density perturbation because we are not dealing with the vertical equation of motion. Consequently, the buoyancy term is absent and the concept of the Boussinesq approximation does not apply.

The final linearized equations are:

$$\frac{\partial u'}{\partial t}+\overline{u}\frac{\partial u'}{\partial x}+\overline{v}\frac{\partial u'}{\partial y}+w'\frac{\partial \overline{u}}{\partial z}=-\frac{1}{\rho_o}\frac{\partial p'}{\partial x}+f(\overline{v}+v')$$
$$\frac{\partial v'}{\partial t}+\overline{u}\frac{\partial v'}{\partial x}+\overline{v}\frac{\partial v'}{\partial y}+w'\frac{\partial \overline{v}}{\partial z}=-\frac{1}{\rho_o}\frac{\partial p'}{\partial y}-f(\overline{u}+u')$$
$$\frac{\partial \theta'}{\partial t}+\overline{u}\frac{\partial \theta'}{\partial x}+\overline{v}\frac{\partial \theta'}{\partial y}+w'\frac{\partial \overline{\theta}}{\partial z}=0$$

5. (25 points) Internal gravity waves can occur only under certain conditions. Given the frequency equation below, specify the PHYSICAL conditions for which the waves will be (a) highest frequency, (b) lowest frequency, and (c) unable to exist.

$$\omega = u_o k \pm \frac{Bk}{\sqrt{k^2 + m^2}}$$

where

$$B^2 = \frac{g}{\bar{\theta}} \frac{d\bar{\theta}}{dz} \, .$$

(a). Linear analysis does not provide information about the amplitude of the waves. However, in terms of frequency, the highest frequency will occur when the buoyancy frequency is large (atmosphere is highly stratified, or potential temperature increases strongly with height), or when the wavenumbers are small (wavelengths are long). Additionally, if the wave is purely horizontal, the particle paths are purely vertical and the frequency is a maximum. We can see this by recalling that the frequency can be written as

$$\omega = u_o k \pm B \cos\theta$$

where θ is the angle between the wavenumber vector and the horizontal. If this angle is 0 degrees, the wave is purely horizontal and the particle motion is parallel to the gravity vector.

(b). Likewise, the waves will be the lowest frequency when B is small (atmosphere is weakly stratified), or when k and m are large (wavelengths are short). If the angle above is 90 degrees, the particle motion is horizontal and thus the frequency is zero (apart from the background wind term).

(c). Internal gravity waves cannot exist when the atmosphere is neutrally stratified, i.e., B=0.

6. (35 points) In class, we showed that the Froude number can be written as follows

$$Fr^2 = \frac{U^2}{H^2 B^2},$$

where U is the large-scale or background wind speed, H is the height of the terrain, and B is the buoyancy frequency. Describe the general behavior of the flow, relative to an isolated terrain feature, under conditions in which U, H and B are large and small in various combinations. More space is available on the next page.

Given the three parameters, it is possible to identify nine different combinations. It is NOT NECESSARY for the students to go through all nine systematically, but rather to see if they have a good understanding overall about the controlling influences on the flow.

In the simplest case, the flow will go AROUND the isolated peak if the Froude number is SMALL. This occurs when the wind speed is low (U small), the stability is high (B large), and the mountain is tall (H large). Conversely, the flow will go OVER the isolated peak if the Froude number is LARGE. This occurs when the wind speed is high (kinetic energy is large and thus the flow can be directed vertically up and over the peak), the stability is low (B is small, thus providing little inhibition to vertical motion owing to weak stratification), and the peak is low (H small), making it easy for flow to go up and over.

One can look at a variety of other combinations. For example, if the stratification is large (B large) but the kinetic energy also is large (U large), the flow may have sufficient inertia to overcome the inhibition to vertical motion caused by stratification to go up and over the peak. Of course, if the peak is high (H large), this will be more difficult than if it is low (H small).

Students can give various combinations like that, and so long as they show solid physical understanding, they should get credit. Again, they needn't do all possible combinations, but I would hope they do the two extreme cases (Fr large and Fr small). Of course, they could also do Fr = 0.