Theory of Thunderstorm Dynamics

Houze sections 7.4, 8.3, 8.5, Refer back to equations in Section 2.3 when necessary.

Bluestein Vol. II section 3.4.6.

Review article "Dynamics of Tornadic Thunderstorms" by Klemp – handout.

A. Equations of Motion

Boussinesq approximated equations (neglecting friction and Coriolis force)

\[
\frac{du}{dt} = -\frac{1}{\rho_0} \frac{\partial p'}{\partial x} \tag{1a}
\]

\[
\frac{dv}{dt} = -\frac{1}{\rho_0} \frac{\partial p'}{\partial y} \tag{1b}
\]

\[
\frac{dw}{dt} = -\frac{1}{\rho_0} \frac{\partial p'}{\partial z} + B \tag{1c}
\]

where buoyancy B includes effects of air density and water loading. The prime is with respect to a horizontally homogeneous base state.

Written in a vector form:
\[
\frac{d\vec{V}}{dt} = -\frac{1}{\rho_0} \nabla p' + B\hat{k} 
\]  

(3)

or

\[
\frac{\partial \vec{V}}{\partial t} = -\vec{V} \cdot \nabla \vec{V} - \frac{1}{\rho_0} \nabla p' + B\hat{k}.
\]  

(4)

Verify it for yourself that

\[
\vec{V} \cdot \nabla \vec{V} = \nabla \left( \frac{\vec{V} \cdot \vec{V}}{2} \right) - \vec{V} \times \left( \nabla \times \vec{V} \right)
\]  

(4a)

Therefore

\[
\frac{\partial \vec{V}}{\partial t} = -\nabla \left( \frac{\vec{V} \cdot \vec{V}}{2} + \frac{p'}{\rho_0} \right) + \vec{V} \times \vec{\omega} + B\hat{k}
\]  

(5)

where \( \vec{\omega} = \nabla \times \vec{V} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ u & v & w \end{vmatrix} \) is the 3D vorticity vector.
B. Vorticity Equation

\[ \vec{\omega} = \left( \frac{\partial w}{\partial y} - \frac{\partial v}{\partial z} \right) \hat{i} + \left( \frac{\partial u}{\partial z} - \frac{\partial w}{\partial x} \right) \hat{j} + \left( \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \right) \hat{k}. \]

Derive 3-D vorticity equation by taking \( \nabla \times (5) \) and recall \( \nabla \times \nabla = 0 \) →

\[ \frac{\partial \vec{\omega}}{\partial t} = \nabla \times (\vec{V} \times \vec{\omega}) + \nabla \times (B\hat{k}) \]  
(6)

1. Development of rotation (vertical vorticity)

To investigate the development of rotation in thunderstorms, look at the vertical component of vorticity \( \zeta = \hat{k} \cdot \vec{\omega} \) →

\[ \frac{\partial \zeta}{\partial t} = \hat{k} \cdot \nabla \times (\vec{V} \times \vec{\omega}) + \hat{k} \cdot \nabla \times (B\hat{k}) = \hat{k} \cdot \nabla \times (\vec{V} \times \vec{\omega}) \]

Note that \( \hat{k} \cdot \nabla \times (B\hat{k}) = 0 \) (verify yourself) therefore buoyancy does not directly generate vertical rotation in thunderstorms! Buoyancy only generates horizontal vorticity which can be tilted into the vertical direction.

Let \( \xi, \eta \) and \( \zeta \) be the \( x, y \) and \( z \) component of vorticity, respectively.
\[
\vec{V} \times \vec{\omega} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ u & v & w \\ \xi & \eta & \zeta \end{vmatrix} = (v_\zeta - w_\eta) \hat{i} + (w_\xi - u_\xi) \hat{j} + (u_\eta - v_\xi) \hat{k}
\]

and

\[
\hat{k} \cdot \nabla \times (\vec{V} \times \vec{\omega}) = \frac{\partial}{\partial x} (w_\xi - u_\xi) - \frac{\partial}{\partial y} (v_\zeta - w_\eta)
\]

\[
\frac{\partial \zeta}{\partial t} = -u \frac{\partial \zeta}{\partial x} - v \frac{\partial \zeta}{\partial y} - w \frac{\partial \zeta}{\partial z} - \zeta \left( \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) + \xi \frac{\partial w}{\partial x} + \eta \frac{\partial w}{\partial y}
\]

(7)

or

\[
\frac{\partial \zeta}{\partial t} = -\vec{V} \cdot \nabla \zeta - \zeta \nabla_{H} \cdot \vec{V} + \vec{\omega}_{H} \cdot \nabla_{H} w
\]

(8)

The last term is called 'tilting' term. It turns horizontal vorticity into the vertical component through differential vertical motion.

Making use of Boussinesq mass continuity equation

\[
\nabla \cdot \vec{V} = \nabla_{H} \cdot \vec{V} + \frac{\partial w}{\partial z} = 0
\]
Eq.(8) can be rewritten as

\[
\frac{\partial \zeta}{\partial t} = -\vec{V} \cdot \nabla \zeta + \zeta \frac{\partial w}{\partial z} + \vec{\omega}_H \cdot \nabla_H w
\]  \hspace{1cm} (9)

**Stretching term** - if \( \zeta > 0 \) and \( w \) increases with height (stretching of air column), the term is positive \( \Rightarrow \) increases in \( \zeta \). Vertical stretching corresponds to horizontal convergence – angular momentum conservation principle!

**Tilting term.**

If shear is in \( u \) only, \( \frac{\partial u}{\partial z} > 0 \), the horizontal vorticity vector points in positive y direction.
On the south side of updraft, \( \frac{\partial w}{\partial y} > 0 \rightarrow \frac{\partial \zeta}{\partial t} = \eta \frac{\partial w}{\partial y} = \frac{\partial u}{\partial z} \frac{\partial w}{\partial y} > 0 \rightarrow \) the tilting creates positively vertical vorticity.

Similarly, on the north side, the tilting due to updraft motion creates negative vertical vorticity.
2. Generation of Horizontal Vorticity (rotation about horizontal axis)

Where the environment has vertical shear, the environment contains horizontal vorticity – the vertical shear is a reservoir of horizontal vorticity. We saw earlier that through tilting of horizontal vortex tubes, horizontal vorticity can be transformed into vertical one – contributing to the thunderstorm rotation.

We pointed earlier that buoyancy does not produce vertical vorticity, what about horizontal vorticity? Yes – it does. Remember horizontal vorticity generation at the gust front where there is strong horizontal buoyancy gradient?

To investigate generation of vorticity about the horizontal, we need the equation for horizontal vorticity. Let's consider the y component of vorticity first:

\[ \eta = \frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}. \]

Take \( \frac{\partial}{\partial z} (Eq.1a) - \frac{\partial}{\partial x} (Eq.1c) \), we obtain (show it yourself)

\[ \frac{d\eta}{dt} = -\frac{\partial B}{\partial x}, \quad (10) \]

therefore (apart from friction), horizontal gradient of buoyancy provides the only source of horizontal vorticity processed by an air parcel.
When the low-level air parcel trajectory has a significant parallel component to the gust front, the vorticity generation by the horizontal gradient can be significant, and the tilting of it into the vertical is believed to contribute significantly to the thunderstorm rotation too.

*Figure 18* Three-dimensional schematic view of a numerically simulated supercell thunderstorm at a stage when the low-level rotation is intensifying. The storm is evolving in westerly environmental wind shear and is viewed from the southeast. The cylindrical arrows depict the flow in and around the storm. The thin lines show the low-level vortex lines, with the sense of rotation indicated by the circular-ribbon arrows. The heavy barbed line marks the boundary of the cold air beneath the storm.
C. Pressure Perturbation Equation

The rotational dynamics with supercell storms has a lot to do with the pressure perturbations created by the air flow. It is this effect that makes supercells special.

Take \( \nabla \cdot (\text{equation of motion}) \), i.e., \( \frac{\partial}{\partial x} (1a) + \frac{\partial}{\partial y} (1c) + \frac{\partial}{\partial z} (1d) \):

1st term:
\[
\frac{\partial}{\partial x} \left[ \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} \right] = -\frac{1}{\rho_0} \frac{\partial^2 p'}{\partial x^2}
\]

2nd term:
\[
\frac{\partial}{\partial y} \left[ \frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z} \right] = -\frac{1}{\rho_0} \frac{\partial^2 p'}{\partial y^2}
\]

3rd term:
\[
\frac{\partial}{\partial z} \left[ \frac{\partial w}{\partial t} + u \frac{\partial w}{\partial x} + v \frac{\partial w}{\partial y} + w \frac{\partial w}{\partial z} \right] = -\frac{1}{\rho_0} \frac{\partial^2 p'}{\partial z^2} + \frac{\partial B}{\partial z}.
\]

Taking derivatives, combining terms and remembering \( \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \), we have

\[
\frac{1}{\rho_0} \nabla^2 p' = -\left[ \left( \frac{\partial u}{\partial x} \right)^2 + \left( \frac{\partial v}{\partial y} \right)^2 + \left( \frac{\partial w}{\partial z} \right)^2 \right] - 2 \left[ \frac{\partial u}{\partial y} \frac{\partial v}{\partial x} + \frac{\partial u}{\partial z} \frac{\partial w}{\partial x} + \frac{\partial v}{\partial z} \frac{\partial w}{\partial y} \right] + \frac{\partial B}{\partial z}.
\]
Now let's partition winds between environmental wind and thunderstorm induced perturbation winds:

\[
\begin{align*}
    u(x,y,z,t) &= \bar{u}(z) + u'(x,y,z,t) \\
    v(x,y,z,t) &= \bar{v}(z) + v'(x,y,z,t) \\
    w(x,y,z,t) &= 0 + w'(x,y,z,t)
\end{align*}
\]

Substituting them into (11), we have

\[
\nabla^2 p' = -\rho_0 \left[ \left( \frac{\partial u'}{\partial x} \right)^2 + \left( \frac{\partial v'}{\partial y} \right)^2 + \left( \frac{\partial w'}{\partial z} \right)^2 + 2 \frac{\partial u'}{\partial x} \frac{\partial v'}{\partial y} + 2 \frac{\partial u'}{\partial z} \frac{\partial w'}{\partial x} + 2 \frac{\partial v'}{\partial z} \frac{\partial w'}{\partial y} \right] - \rho_0 \left[ 2 \frac{\partial \bar{u}}{\partial z} \frac{\partial w'}{\partial x} + 2 \frac{\partial \bar{v}}{\partial z} \frac{\partial w'}{\partial y} \right] + \rho_0 \frac{\partial B}{\partial z}
\]

(12)

It's an elliptic diagnostic equation for pressure $p'$.

Dividing the total perturbation pressure into

\[
p' = p'_{\text{dyn}} + p'_{\text{B}}
\]

\[
= p'_{\text{L}} + p'_{\text{NL}} + p'_{\text{B}}
\]

2nd term "fluid extension term" + shear term last term
Each part is attributed to certain terms on the right hand side of Eq.(12).

**Applications:**

1. Updraft enhancement in rotating thunderstorms and cell splitting.

We saw earlier that a strong updraft in an environment of significant vertical shear produces a pair of counter-rotating vortices. Consider \( p'_{NL} \) term only. It can be verified that the 3 shear terms can be written in the following form:

\[
2 \frac{\partial u'}{\partial y} \frac{\partial v'}{\partial x} + 2 \frac{\partial u'}{\partial z} \frac{\partial w'}{\partial x} + 2 \frac{\partial v'}{\partial z} \frac{\partial w'}{\partial y} \\
= \frac{1}{2} \left[ \left( \frac{\partial v'}{\partial x} + \frac{\partial u'}{\partial y} \right)^2 + \left( \frac{\partial v'}{\partial z} + \frac{\partial w'}{\partial y} \right)^2 + \left( \frac{\partial u'}{\partial z} + \frac{\partial w'}{\partial x} \right)^2 \\
- \left( \frac{\partial v'}{\partial x} - \frac{\partial u'}{\partial y} \right)^2 - \left( \frac{\partial v'}{\partial z} - \frac{\partial w'}{\partial y} \right)^2 - \left( \frac{\partial u'}{\partial z} - \frac{\partial w'}{\partial x} \right)^2 \right]
\]

If we assume pure rotation (no div, deformation) and ignore extension terms (i.e., look at the effect of rotation only), then

\[
\nabla^2 p'_{NL} = \rho_0 \left( \frac{\partial v'}{\partial x} - \frac{\partial u'}{\partial y} \right)^2 = \frac{\rho_0}{2} \varsigma^2.
\]

Since the left hand side is a Laplacian operator, \( \nabla^2 p'_{NL} \propto -p'_{NL} \), therefore
\[ p'_{NL} \propto -\zeta^2. \]  

→ **Both cyclonic and anticyclonic rotation produces negative pressure perturbation.** The low pressure center is actually required to have a PGF that balances the centrifugal force!

Negative \( p'_{NL} \) is largest where rotation strongest, which is usually at the mid-levels of thunderstorms -- \( p'_{NL} \) "low" there up to 2-4 mb.

Earlier figure shows that because of tilting, **vertical rotation is strongest at the flanks of updraft, and negative \( p' \) at the mid-levels creates an upward pressure gradient force there that promotes new updrafts there -- a dynamic cause for cell splitting.**

Now, let's consider the fluid extension terms,

\[
\nabla^2 p' = -\rho_0 \left[ \frac{\partial (u')^2}{\partial x} + \frac{\partial (v')^2}{\partial y} + \frac{\partial (w')^2}{\partial z} + ... \right] < 0
\]

\[
p' \propto \rho_0 \left[ \frac{\partial (u')^2}{\partial x} + \frac{\partial (v')^2}{\partial y} + \frac{\partial (w')^2}{\partial z} + ... \right] > 0
\]

So **positive \( p' \) is largest where stretching is largest.** This occurs at the low and high levels.
So we have

\[ p'_\text{NL} > 0 \quad \text{H} \]

\[ p'_\text{NL} < 0 \quad \text{L} \]

\[ \uparrow \quad \text{vertical PGF} \]

\[ p'_\text{NL} > 0 \quad \text{H} \]

Therefore the **nonlinear pressure perturbation due to shear and stretching creates additional upward lifting (pressure gradient) force at the lower atmosphere that enhances the updraft beyond that based on buoyancy!** Therefore, supercell storms tend to be stronger than regular storms, given the same amount of CAPE.

Rule of thumb: 1mb VPGF over 1 km ~ same forcing as 3°C of buoyancy.
Figure 3.23  Schematic diagram depicting how a typical vortex line (streamline of three-dimensional vorticity vector) contained within (westerly) environmental shear is deformed as it interacts with a convective cell (viewed from the southeast). Direction of cloud-relative airflow (cylindrical arrows); vortex lines (solid lines), with the sense of rotation indicated by circular arrows; the forcing influences that promote new updraft and downdraft growth (shaded arrows); regions of precipitation (vertical dashed lines). (a) Initial stage: Vortex line loops into the vertical as it is swept into the updraft. (b) Splitting stage: Downdraft forming between the splitting updraft cells tilts vortex line downward, producing two vortex pairs. Boundary of the cold-air spreading out beneath the storm (cold-front symbol) at the
Figure 3.22  Vertical cross section of acceleration induced by vertical perturbation-pressure force for the linear part of the wind field ($-\partial \pi_l / \partial z$ (top) and the nonlinear part of the wind field ($-\partial \pi_{nl} / \partial z$) (bottom) in a numerical simulation 10 min after storm initiation. The environmental wind profile has a straight-line hodograph. Contours plotted every 0.004 m s$^{-2}$ (from Rottuno and Klemp, 1982). (Courtesy of the American Meteorological Society)
2. Preferred enhancement of right-moving or left-moving storm

Consider the linear $p'_{\text{dyn}}$ term:

\[ \nabla^2 p' = -\rho_0 \left[ 2 \frac{\partial \bar{u}}{\partial z} \frac{\partial w'}{\partial x} + \frac{\partial \bar{v}}{\partial z} \frac{\partial w'}{\partial y} \right] = -2\rho_0 \frac{\partial \bar{V}}{\partial z} \cdot \nabla w' \]

so \[ p' \propto \frac{\partial \bar{V}}{\partial z} \cdot \nabla w'. \] (14)

For unidirectional shear

\[ p' \propto \frac{\partial \bar{u}}{\partial z} \frac{\partial w'}{\partial x} > 0 \text{ on the west/upshear flank of updraft} \]
\[ p' \propto \frac{\partial u}{\partial z} \frac{\partial w'}{\partial x} < 0 \] on the east/downshear flank of updraft and \( p' \) largest at the mid-levels.

\[ p' > 0 \quad p' < 0 \]

→ new cell growth on the downshear flank

If the hodograph clockwise curved,

then we also have to consider \( \frac{\partial v}{\partial z} \frac{\partial w'}{\partial y} \) term in (14).
At low levels, \( \frac{\partial v}{\partial z} \frac{\partial w'}{\partial y} \), produces

At the mid-levels, \( \frac{\partial u}{\partial z} \frac{\partial w'}{\partial x} \) produces

At the upper levels \( \frac{\partial v}{\partial z} \frac{\partial w'}{\partial y} \) produces

\( \rightarrow \) there is a upward vertical PGF on the right flank of the storm (downward PGF on the left flank) \( \rightarrow \) new cell growth is enhanced to the right, rotating updraft becomes a 'right mover'.
Figure 3.21 Schematic diagram illustrating the pressure and vertical vorticity perturbations arising as an updraft interacts with an environmental vertical wind shear vector that (a) does not change direction with height and (b) turns clockwise with height. The high (H) to low (L) horizontal pressure-gradient forces parallel to the shear vectors (flat arrows) are labeled along with the preferred location of cyclonic (+) and anticyclonic (−) vorticity. Shaded arrows depict orientation of resulting vertical pressure-gradient forces (from Klemp, 1987; adapted from Rotunno and Klemp, 1982). (Courtesy of the American Meteorological Society)
Figure 3.24 Acceleration induced by vertical perturbation-pressure gradient force at 1.5 km in a numerical simulation 10 min after storm initiation for (a) the linear part of the wind field for a straight-line hodograph; (b) the nonlinear part of the wind field for a straight-line hodograph; (c) the linear part of the wind field for a clockwise-turning hodograph; (d) the nonlinear part of the wind field for a clockwise-turning hodograph. Contours plotted every 0.004 m s\(^{-2}\); (e) hodographs used in the simulations; clockwise-turning hodograph indicated by solid line; counter-clockwise-AFL (from Rutledge and Klemp, 1982). (Courtesy of the American Meteorological Society.)
Figure 3.25 (Top) Radar-echo history of a splitting storm observed in south-central Oklahoma. Radar reflectivity of 10 dBZ (solid lines); radar reflectivity in excess of 40 dBZ (stippled regions). Times adjacent to each outline are CST. (Bottom) Hodograph representative of the storm's environment. Heights in km AGL. Motion of the right-moving (RM) and left-moving (LM) cells (from Weisman and Klemp, 1986; adapted from Burgess, 1974). (Courtesy of the American Meteorological Society)
Summary:

The nonlinear-shear effect that promotes new or continued cell growth on the flanks of (alongside) the old cell;

The linear effect of tilting biases the cell movement toward the right (left) if the environmental hodograph is curved in a clockwise (counterclockwise) manner;

Unidirectional shear promotes storms that split, with each member of the split pair having components of motion normal to the shear vector and opposite to each other.

New buoyant updrafts form off the axis of the shear because upward-directed perturbation pressure gradients induce upward accelerations there and lift air to its LFC.

Owing to the low-level convergence associated with upward-moving air, vorticity increases through the stretching of the existing vorticity and is advected upward by the updraft:

Right movers tend to develop cyclonic rotation, while left movers tend to develop anti-cyclonic rotation. The cyclonic vorticity produced in storms that grow in an environment of clockwise turning shear is not due to the Earth's rotation.
Thunderstorm Dynamics

Helicity and Hodographs and their effect on thunderstorm longevity

Bluestein Vol II. Pages 471-476.

Cross-stream and streamwise vorticity

In the following figure, **storm-relative flow is in the same direction as the vertical shear vector** $\to$ **flow is normal to the vorticity vector**, therefore the vorticity is **cross-stream**.

Assume potential temperature is conserved below the cloud and $\theta_e$ is conserved in the cloud layer. Initially the $\theta$ (or $\theta_e$) surfaces are flat. A 'bump' forms when there is an updraft. The storm-relative inflow turns upward on the upstream side of the bump and downward on the downstream side. Corresponding on the right side of the bump is upward titled vortex tube and the left side downward tilted vortex tube, corresponding to max/min vorticity. In this situation, the **w and $\omega_r$, are not correlated, there is no streamwise vorticity**.
In the next situation, storm-relative flow is normal to the vertical-shear vector, the flow is parallel to the horizontal vorticity vector.

In this case, the max w coincides with max $\omega_H$, and $w$ and $\omega_H$ are strongly (positively) correlated. There is large streamwise vorticity.

Figure 3.26 (Top) Effect of localized vertical displacement “peak” (i.e., “hump” in isentropic surface) on vortex lines when mean vorticity and mean storm-relative flow $\vec{v} - \vec{c}$ are perpendicular (purely crosswise vorticity). The peak draws up loops of vortex lines (shown slightly above, instead of in, the surface for clarity), giving rise to cyclonic vorticity on the right side of the peak (relative to shear vector $\vec{s}$) and anticyclonic vorticity on the left side. (Solenoidal effects (not included in figure) actually divert the vortex lines around the right (left) side of a warm (cool) peak, relative to the mean vorticity vector, but do not alter the result.) Environmental flow across expanding peak produces maximum updraft upstream (storm-relative frame) of peak owing to up-slope component of vertical velocity. In this case, there is no correlation between vertical velocity and vertical vorticity since centers of the $w'$ field lie in different quadrants of peak from those of $\zeta'$ field. (Bottom) As in top panel, but for other extreme when vorticity is purely streamwise (i.e., $\omega$ is parallel to $\vec{v} - \vec{c}$). Here, the up-slope (down-slope) side of the peak is also the cyclonic (anticyclonic) side, and vertical velocity and vertical velocity are positively correlated (from Davies-Jones, 1984). (Courtesy of the American Meteorological Society)
The following figure shows the horizontal vorticity $\tilde{\omega}_H$ at various points on a hodograph.

What causes vorticity to be crosswise??

- when storm motion vector lies on hodograph!
In the above example, the hodograph is a straight line and the shear is undirectional (the flow is not unidirectional, however). If the storm-motion vector $\vec{C}$ lies on the hodograph, then the storm-relative flow is always parallel to the shear vector therefore perpendicular to the vorticity vector $\vec{\omega}$. The vorticity is crosswise.

For the above example, if the storm splits into the right and left mover. The storm motion vector for the right mover $\vec{R}$ now lies on the right side of the hodograph. The following example shows that in this situation, one gets significant streamwise vorticity.

**What hodograph causes large streamwise $\vec{\omega}$?**

- when $\vec{C}$ lies to the right of hodograph!
We see, for example at point 2, that the storm-relative velocity vector $\vec{V}_{r2}$ is essentially parallel to the shear vorticity vector.

We can compute the component of vorticity $\vec{\omega}$ in the direction of the storm-relative velocity $\vec{V} - \vec{C}$ as

$$\vec{\omega}_s = \frac{(\vec{V} - \vec{C}) \cdot \nabla \times \vec{V}}{|\vec{V} - \vec{C}|}$$

which we call the **streamwise vorticity**.

The numerator,
\[ H = (\mathbf{V} - \mathbf{C}) \cdot \nabla \times \mathbf{V} \]

is known as \textit{helicity}.

In the case of streamwise vorticity, which is present only when the wind direction changes with height (again, neglecting the contribution from vertical motion's horizontal gradient), one can visualize the flow as being helical; a good mental image is a passed football rotating in a "spiral." Hence, the term \textit{helicity} is associated directly with streamwise vorticity.

Schematic showing how the superposition of horizontal vorticity \((\mathbf{\Omega}_h)\) parallel to the horizontal flow \((\mathbf{V}_h)\) produces a helical flow.

Another quantity, called the \textbf{coefficient of streamwise vorticity},

\[ \text{RH} = \frac{(\mathbf{V} - \mathbf{C}) \cdot \nabla \times \mathbf{V}}{|\mathbf{V} - \mathbf{C}||\nabla \times \mathbf{V}|} \]
is often used too, and it is also called **relative helicity**.

Davies-Jones has shown that the correlation coefficient for storm vertical vorticity and storm vertical velocity is approximately proportional to the environmental relative helicity (calculated based environmental shear and horizontal vorticity).

**Why large helicity is beneficial for storms?**

Dough Lilly suggested that the longevity of supercell storms is due to their large helicity. Consider the 3D vorticity equation that we derived earlier:

\[
\frac{\partial \vec{\omega}}{\partial t} = \nabla \times (\vec{V} \times \vec{\omega}) + \nabla \times (B \hat{k})
\]

If the storm-relative velocity vector \( \vec{V} \) points in the same direction as the vorticity vector \( \vec{\omega} \), then

\[
\vec{V} \times \vec{\omega} = 0,
\]

therefore the first term on the right hand side is zero – this term give rises to advection, stretching and tilting terms as we say earlier. When it is zero, the only source of vorticity is the buoyancy production term, which contributes only to the horizontal vorticity, not vertical component of vorticity. In this situation, vertical vorticity is conserved – therefore rotation can be maintained effectively. We see more clearly from the following.

Making use of vector identity

\[
\nabla \times (\vec{a} \times \vec{b}) = \vec{a}(\nabla \cdot \vec{b}) - \vec{b}(\nabla \cdot \vec{a}) - (\vec{a} \cdot \nabla)\vec{b} + (\vec{b} \cdot \nabla)\vec{a}
\]
letting $\vec{a} \equiv \vec{V}$, $\vec{b} \equiv \vec{\omega}$, and noting $\nabla \cdot \vec{\omega} = 0$, we can obtain

$$0 = \nabla \times (\vec{V} \times \vec{\omega}) = \vec{V} (\nabla \cdot \vec{\omega}) - \vec{\omega} (\nabla \cdot \vec{V}) - (\vec{V} \cdot \nabla) \vec{\omega} + (\vec{\omega} \cdot \nabla) \vec{V} \Rightarrow$$

$$\vec{V} \cdot \nabla \vec{\omega} = -\vec{\omega} \cdot \nabla \vec{V} + \vec{\omega} \cdot \nabla \vec{V}$$

The left hand side is the vorticity advection, the right hand side are the stretching term (related to divergence) and tilting term. The equations say that the effect of advection is cancelled by stretching and tilting. Nonlinear advection is a source of energy dissipation, through what's called the energy cascade to smaller scales due to nonlinear wave-modes interaction. The cascade to smaller scales eventually causes energy dissipation.

Thus if a storm becomes a right-mover due to perturbation pressure dynamics, the magnitude of the correlation between $w'$ and $\omega_s$ tends to be positive and large, and causes larger streamwise vorticity. The right-mover, from this view point, is also favored for clockwisely curved hodographs - because the storm motion vector of the right mover is further away from the hodograph therefore the storm-relative wind, especially that component that is parallel to the horizontal vorticity, is larger.

**However**, $(\vec{V} - \vec{C}) \cdot \nabla \times \vec{V}$ is not Galilean invariant, i.e., it is dependent on the coordinate system (following the storm motion) chosen. Therefore there is no single value of helicity for a given sounding (unlike CAPE) – it completely depends on value of $\vec{C}$.

It is believed that the storm-relative helicity in the lowest two or three kilometers of the atmosphere is most relevant to the likelihood of supercell behavior with storms in that environment.

Therefore, helicity integrated over the lowest 3 km, i.e.,
is often calculated and used as a guidance for thunderstorm forecast. Here we assume the vorticity is given by the environmental wind vector $\vec{V}$.

To do that one also has to estimate the storm motion vector $\vec{C}$. A crude way is to use the pressure-weighted mean wind in the lowest 5 to 6 km.

It turns out that the vertical integrated helicity is equal to minus twice the signed area swept out by the storm-relative winds between the surface and height $Z$, i.e.

See detailed explanation in McCaul et al writeup. Therefore, the further is the tip of storm-motion vector from the hodograph, usually the larger is the helicity.
Examples of hodographs from environments that produced significant tornado outbreaks in Oklahoma are given below.

FSI 1500 CST
EMC 1620 CST
20 MAY 1977

FSI 1900 CST
11 MAY 1982
Early results indicate that for tornado producing environment, H ranges from approximately 150 m$^2$ s$^{-2}$ to upwards of 1000 m$^2$s$^{-2}$. Davies-Jones (1990) examined the results of 28 tornado cases with the following categories for H:

- $150 < H < 299$ weak tornadoes
- $300 < H < 449$ strong tornadoes
- $H > 450$ violent tornado

It is important to point out that helicity will not determine whether or not storms will develop, but instead indicates how a particular storm (or storms) may evolve given the ambient shear.

**Strongly veering hodograph versus straight hodographs**

At first glance, tornadic storms would be most likely only when the hodograph shows vertical shear that veers strongly with height, such storms are also possible when the hodograph is relatively straight.

Considerable streamwise vorticity may be present with a straight hodograph if storm motions lie significantly to the right of the hodograph. This would occur when split cells move sideways away from the “steering level” flow that originally lies on the straight hodograph.

An example of a hodograph which was approximately straight from 2 km to 11 km of altitude, along with the observed storm motions. (L = left-mover, R = right-mover) for a splitting storm pair, are shown below. Also shown are the tracks of splitting storms observed by radar, and the corresponding tracks of storms simulated numerically for similar environmental conditions.
FIG. Proximity hodograph for the Union City, Oklahoma, splitting storm.
FIG. The observed and modeled storm development on 2 April 1964, which has similar environment as the Union City OK storm. The storms are labeled and are several times the contoured regions are stippled for better visualization. (From Wilhelmson and Klemp. 1981.)
Summary - Why do we look at helicity?

- Given a particular environment, certain aspects of the ambient flow may enhance the energetics and longevity of storms that develop.

- Helicity provides yet another method to deduce and/or ascertain information pertaining to the internal dynamics of severe storms.

- Helicity is implicitly coupled to both kinetic energy and enstrophy, consequently, disturbance energy calculations are possible via the "helicity equations".

- Theory suggests the suppression of the inertial energy cascade as a result of the helical nature of a storm.

- From models such as the highly idealized Beltrami flow, which describes a purely helical flow, we can deduce:
  
  a. The rotational characteristics of mesocyclone
  b. The dynamic pressure distribution in and around rotating storms

- Helicity is proportional to the low-level streamwise vorticity, and if taken as a storm-relative quantity is proportional to the strength of the low-level storm inflow.

- Helicity explicitly accounts for storm motion.

- Helicity can be easily calculated from the area on a hodograph diagram (more later).
Finally, we note that because the storm-relative (environmental) helicity depends on how far the storm-motion vector is from the hodograph curve, one can plot 'helicity contours' on a hodograph. It tells us that if the storm-motion vector falls in certain areas on a hodograph, you get certain amount of helicity. The following is an example from Droegemeier et al (1993).

**HALF4**

![Wind hodograph for simulation HALF4. The solid square is the density-weighted mean wind in the layer 0–6 km, and the stars represent the motion of the initial storm, based on tracking the updraft maximum at $z = 3$ km, over the periods shown. Concentric circles are separated by 2.5 m s$^{-1}$, and hodograph labels are altitude (km). Also shown are contours of the storm-relative environmental helicity (parallel thin solid lines) computed following Davies-Jones et al. (1990).](image)

FIG. 10. Wind hodograph for simulation HALF4. The solid square is the density-weighted mean wind in the layer 0–6 km, and the stars represent the motion of the initial storm, based on tracking the updraft maximum at $z = 3$ km, over the periods shown. Concentric circles are separated by 2.5 m s$^{-1}$, and hodograph labels are altitude (km). Also shown are contours of the storm-relative environmental helicity (parallel thin solid lines) computed following Davies-Jones et al. (1990).
Bulk Richardson Number

The quantitative relations between environmental shear and thermal instability and resulting storm type have been studied with numerical models (Weisman and Klemp, 1982; 1984). The results indicate that a parameter known as the bulk Richardson number (BRN), defined by:

$$BRN = \frac{CAPE}{\frac{1}{2} \bar{U}^2}$$

is a good predictor of storm type.

In the above formula, CAPE is the convective available potential energy (positive area) of the sounding, and $U = \bar{u}_{6000} - \bar{u}_{500}$ is a measure of wind shear below 6 km level, and $\bar{u}_{6000}$ and $\bar{u}_{500}$ are the density-weight mean wind in the lowest 6 km and 500m layers of the atmosphere, respectively (Weisman and Klemp, 1982).

According to Weisman and Klemp,

- When BRN $\geq$ 40, storms are likely to be multicell, and
- When BRN $\leq$ 40, storms may be supercell
- When BRN < 10 with unidirectional shear, storms may be suppressed by the excessive shear, although with curved hodographs this suppression is less noticeable.

It should be noted that a shortage of CAPE in a sounding may be overcome if mesoscale or synoptic scale dynamical lifting is sufficiently strong. However, a shortage of shear (equivalent to weak low-level storm inflow) or an unfavorable hodograph shape are much more difficult to compensate for.
Figure 15.18. Richardson number R as calculated for a series of documented storms. Model results are summarized at the top of the figure. S1, S2, ..., S9 represent supercell storms; M1, M2, ..., M5 represent multicell storms; TR1, TR2, ..., TR4 represent tropical cases. (Adapted from Weisman and Klemp, 1982.)
Summary – Roles of shear and CAPE

- Strong vertical shear and a veering or straight hodograph in an environment of veering winds are important in setting the stage for supercell convection and possible tornadoes.

- Large CAPE is also desirable, although this requirement may be relaxed if strong lifting is present.

- Absence of shear, however, is an obstacle to supercell development that is more difficult to overcome.

- Nevertheless, hodograph structure is very sensitive to changes in the wind field, and many mesoscale regions of favorable shear are never sampled by the existing rawinsonde network.

- It is therefore imperative that the forecaster use all available data - surface obs, upper air obs, wind profilers, radar data and satellite images – in assessing the risks of severe weather in the thunderstorm season.

Reference:


The Influence of Helicity on Numerically Simulated Convective Storms

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ABSTRACT

A three-dimensional numerical cloud model is used to investigate the influence of storm-relative environmental helicity (SREH) on convective storm structure and evolution, with a particular emphasis on the identification of ambient shear profiles that are conducive to the development of long-lived, strongly rotating storms. Eleven numerical simulations are made in which the depth and turning angle of the ambient vertical shear vector are varied systematically while maintaining a constant magnitude of the shear in the shear layer. In this manner, an attempt is made to isolate the effects of different environmental helicities on storm morphology and show that the SREH and bulk Richardson number, rather than the mean shear in the low levels, determine the rotational characteristics and morphology of deep convection.

The results demonstrate that

- **Storms forming in environments characterized by large SREH are longer-lived than those in less helical surroundings.**

- **The storm-relative winds in the layer 0–3 km must, on average, exceed 10 m s⁻¹ over most of the lifetime of a convective event to obtain supercell storms.**

- **The correlation coefficient between vertical vorticity \( \zeta \) and vertical velocity \( w \), which (according to linear theory of dry convection) should be proportional to the product of the normalized helicity density, NHD (i.e., relative helicity), and a function involving the storm-relative wind speed, has the largest peak**
values (in time) in those simulated storms exhibiting large SREH and strong storm-relative winds in the low levels.

- Even when the vorticity is predominantly streamwise in the storm-relative framework, giving a normalized helicity density near unity (as is the case in many of these simulations), **significant updraft rotation and large w-ζ correlation coefficients do not develop and persist unless the storm-relative winds are sufficiently strong.**

- The correlation coefficient between w and ζ′ based on linear theory is found to be a significantly better predictor of net updraft rotation than the bulk Richardson number (BRN) or the BRN shear, and slightly better than the 0-3-km SREH.

- Both the theoretical correlation coefficient and the SREH are based on the motion of the initial storm after its initially rapid growth. Linear theory also predicts correctly the relative locations of the buoyancy, vertical velocity, and vertical vorticity extrema within the storms after allowance is made for the effects of vertical advection.

- In predicting the maximum vertical vorticity both above and below 1.14 km, rather than the actual w-ζ correlation, the 0-3-km SREH performs slightly worse than the BRN.

- The correlation coefficient, SREH, and BRN all do a credible job of predicting storm type.

- Thus, it is recommended that operational forecasters use the BRN to predict storm type because it is independent of storm motion, and the SREH to characterize the rotational properties of storms once their motions can be established.

- Finally, the ability of the NHD to characterize storm type and rotational properties is examined. Computed using the storm-relative winds, the **NHD shows little ability to predict storm rotation (i.e., maximum w-ζ**
correlation and maximum vertical vorticity), because it neglects the magnitudes of the vorticity and storm-relative wind vectors.

- Histograms of the disturbance NHD show a distinct bias toward positive values near unity for supercell storms, indicating an extraction of helicity from the mean flow by the disturbance, and only a slight bias for multicell storms.