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METR 3113, Atmospheric Dynamics I
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Review of Taylor Series

Given a function F(x), one can write a Taylor series expansion for F about the point x_o in the following ways.

Form #1

$$F(x) = F(x_o) + \left.\frac{\partial F}{\partial x}\right|_{x_o}(x - x_o) + \left.\frac{\partial^2 F}{\partial x^2}\right|_{x_o}\frac{(x - x_o)^2}{2!} + HigherOrderTerms$$

Form #2 (most commonly used in fluid mechanics and meteorology)

Here, we simply replace x by $x_o + \Delta x$ in the above, where Δx is a small increment about the point:

$$F(x_o + \Delta x) = F(x_o) + \left.\frac{\partial F}{\partial x}\right|_{x_o}(\Delta x) + \left.\frac{\partial^2 F}{\partial x^2}\right|_{x_o}\frac{(\Delta x)^2}{2!} + \left.\frac{\partial^3 F}{\partial x^3}\right|_{x_o}\frac{(\Delta x)^3}{3!} + HigherOrderTerms$$

Points to Note

1. One can write a Taylor series as a function of 2 variables in the following manner:

$$F(x_o + \Delta x, y_o + \Delta y) = F(x_o, y_o) + \left(\Delta x\frac{\partial}{\partial x} + \Delta y\frac{\partial}{\partial y}\right)F(x_o, y_o) + \frac{1}{2!}\left(\Delta x\frac{\partial}{\partial x} + \Delta y\frac{\partial}{\partial y}\right)^2 F(x_o, y_o) + HigherOrderTerms$$

Note that the ( ) terms raised to a power are actually operators and thus involve mixed derivatives. For example, we can expand the last term as

$$\left(\Delta x\frac{\partial}{\partial x} + \Delta y\frac{\partial}{\partial y}\right)^2 = (\Delta x)^2\frac{\partial^2}{\partial x^2} + 2\Delta x\Delta y\frac{\partial^2}{\partial x\partial y} + (\Delta y)^2\frac{\partial^2}{\partial y^2}$$

2. One can use positive and negative increments in Form #2, with the sign attached to the increment itself, i.e., +Δx and -Δx. This allows us to write two series in one:

$$F(x_o \pm \Delta x) = F(x_o) \pm \left.\frac{\partial F}{\partial x}\right|_{x_o}(\Delta x) + \left.\frac{\partial^2 F}{\partial x^2}\right|_{x_o}\frac{(\Delta x)^2}{2!} \pm \left.\frac{\partial^3 F}{\partial x^3}\right|_{x_o}\frac{(\Delta x)^3}{3!} + HigherOrderTerms$$

By subtracting or adding the series, we can arrive at various approximations for derivatives. For example, subtracting and solving for $\partial F / \partial x$ gives

$$\left.\frac{\partial F}{\partial x}\right|_{x_o} = \frac{F(x_o + \Delta x) - F(x_o - \Delta x)}{2\Delta x} + HigherOrderTerms$$