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METR 3113 – Atmospheric Dynamics I
Fall 2016

Problem Set #2

Distributed Friday, 2 September 2016
Due Monday, 12 September 2016

INSTRUCTIONS: For each of the problems below, apply all 6 steps in the problem-solving handout. Pay close attention to neatness, and describe your work at each step of the solution process.

1. Polar coordinates. Assuming the following equation for a particular form of the Archimedes spiral in polar coordinates:

$$r = \frac{\theta}{2\pi},$$

calculate Cartesian slopes of the tangent lines to the Archimedes spiral at

$$\theta = \frac{\pi}{4},\ \theta = \frac{\pi}{2},\ \theta = \frac{3\pi}{4},\ \theta = \pi,\ \theta = 2\pi, \text{ and } \theta = 2200\pi.$$

What would be the value of *r* corresponding to $\theta = 2200\pi$? It would be useful to draw a schematic plot of the Archimedes spiral and look up on the Internet information about this remarkable curve and other related spirals.

2. Polar coordinates. Calculate the polar coordinates (r, θ) of the following points in the 2-D Cartesian system (X, Y):

(24, 10), (-10, 24), (-10, -24), and (24, -10),

assuming usual conventions regarding collocation of Cartesian and polar systems, and the choice of polar angle.

3. Coordinate reflection. Show that reflection of a point across a line passing through the origin at an angle α with the X axis transforms the point coordinates (x, y) into (x', y') with the coordinates related as

$$x' = x\cos 2\alpha + y\sin 2\alpha,$$
$$y' = x\sin 2\alpha - y\cos 2\alpha,$$

or

$$(x', y') = (x\cos 2\alpha + y\sin 2\alpha,\ x\sin 2\alpha - y\cos 2\alpha).$$

4. Ekman Spiral. Consider a local Cartesian coordinate system (*X, Y, Z*) in the northern hemisphere where *X* Cartesian coordinate axis is assumed to be directed eastward and *Y* Cartesian coordinate axis is assumed to be directed northward (conventional geographic system).

a. Calculate and plot Ekman-model *x* and *y* wind components, $u(z)$ and $v(z)$, and corresponding wind hodograph (Ekman spiral) for the range of *z* from 0 to 4000 m in the above specified coordinate system. Model parameters: *x* and *y* geostrophic wind components are $U_g = -5\ \mathrm{m\,s^{-1}}$ and $V_g = 8\ \mathrm{m\,s^{-1}}$, respectively; Coriolis parameter is $f = 10^{-4}\ \mathrm{s^{-1}}$; and eddy diffusivity is $k = 50\ \mathrm{m^2 s^{-1}}$. Evaluate the planetary boundary layer depth scale.

b. Calculate and plot Ekman-model wind components and corresponding wind hodograph (Ekman spiral) for the range of *z* from 0 to 4000 m and with the same values of f and k as in **a**, but in the Cartesian coordinate system with the *X* axis (denote it as *X*') directed along the geostrophic wind vector from case **a**.

c. Calculate and plot Ekman-model wind components, and corresponding Ekman spiral for the range of *z* from 0 to 4000 m and with the same values of f and k as in **a**, but in the Cartesian coordinate system with the *X* axis (denote it as *X*'') directed along the near-surface wind from case **a**.

5. Ekman Spiral. What is the value of the angle between the near-surface wind and geostrophic wind in the Ekman model of the atmospheric boundary layer flow?

6. Relating Cartesian to polar coordinates. In Lecture 5, we derived relationships between Cylindrical and polar coordinates. Show that (see Lecture 5) the Cartesian slope of the tangent line to a polar curve r(θ) at any point is given by

$$\frac{dy}{dx} = \frac{\frac{dr}{d\theta}\sin\theta + r\cos\theta}{\frac{dr}{d\theta}\cos\theta - r\sin\theta}$$

7. Polar coordinates. Find the distance between points (1, π/2), and (1, -π/2) in 2-D polar coordinates.