Lecture 37. November 30, 2016

Topic: Gradient wind classification for the Southern Hemisphere.

Reading: Holton and Hakim Section 3.2.

1. General solution

Solution of the gradient wind equation (see Class 39),

$$\frac{V^2}{R} + fV = -\frac{\partial \Phi}{\partial n},$$

is

$$V = -\frac{fR}{2} \pm \left(\frac{f^2 R^2}{4} - R\frac{\partial \Phi}{\partial n} \right)^{1/2}.$$

In terms of geostrophic wind velocity, $V_g = -\frac{1}{f}\frac{\partial \Phi}{\partial n}$, it may be written as

$$V = -\frac{fR}{2} \pm \left(\frac{f^2 R^2}{4} + fRV_g \right)^{1/2},$$

where V (the gradient wind speed) is a real and non-negative, by definition, quantity.

2. Particular solutions

Particular gradient wind solutions for the Southern Hemisphere (where $f < 0$) are the following (compare to solutions for the Northern Hemisphere obtained in Class 40).

1. $\frac{\partial \Phi}{\partial n} > 0$, $V_g > 0$.

1a. $R > 0$ and $V = -\frac{fR}{2} + \left(\frac{f^2 R^2}{4} - R\frac{\partial \Phi}{\partial n} \right)^{1/2} = -\frac{fR}{2} + \left(\frac{f^2 R^2}{4} + fRV_g \right)^{1/2}$ > 0: physical.

1b. $R > 0$ and $V = -\frac{fR}{2} - \left(\frac{f^2 R^2}{4} - R\frac{\partial \Phi}{\partial n} \right)^{1/2} = -\frac{fR}{2} - \left(\frac{f^2 R^2}{4} + fRV_g \right)^{1/2}$ > 0: physical.

1c. $R < 0$ and $V = -\frac{fR}{2} + \left(\frac{f^2 R^2}{4} - R\frac{\partial \Phi}{\partial n} \right)^{1/2} = -\frac{fR}{2} + \left(\frac{f^2 R^2}{4} + fRV_g \right)^{1/2}$ > 0: physical.

1d. $R < 0$ and $V = -\frac{fR}{2} - \left(\frac{f^2 R^2}{4} - R\frac{\partial \Phi}{\partial n} \right)^{1/2} = -\frac{fR}{2} - \left(\frac{f^2 R^2}{4} + fRV_g \right)^{1/2}$ < 0: unphysical.

2. $\frac{\partial \Phi}{\partial n} < 0$, $V_g < 0$.

2a. $R > 0$ and $V = -\frac{fR}{2} + \left(\frac{f^2R^2}{4} - R\frac{\partial \Phi}{\partial n} \right)^{1/2} = -\frac{fR}{2} + \left(\frac{f^2R^2}{4} + fRV_g \right)^{1/2}$ > 0: physical.

2b. $R > 0$ and $V = -\frac{fR}{2} - \left(\frac{f^2R^2}{4} - R\frac{\partial \Phi}{\partial n} \right)^{1/2} = -\frac{fR}{2} - \left(\frac{f^2R^2}{4} + fRV_g \right)^{1/2}$ < 0: unphysical.

2c. $R < 0$ and $V = -\frac{fR}{2} + \left(\frac{f^2R^2}{4} - R\frac{\partial \Phi}{\partial n} \right)^{1/2} = -\frac{fR}{2} + \left(\frac{f^2R^2}{4} + fRV_g \right)^{1/2}$ < 0: unphysical.

2d. $R < 0$ and $V = -\frac{fR}{2} - \left(\frac{f^2R^2}{4} - R\frac{\partial \Phi}{\partial n} \right)^{1/2} = -\frac{fR}{2} - \left(\frac{f^2R^2}{4} + fRV_g \right)^{1/2}$ < 0: unphysical.

2. Force balance and circulation types for physical flow cases

Now consider force balances and flow patterns corresponding to physical flow cases from above. **Note** that in the Southern Hemisphere, where the angular momentum of planetary motion is negative, the flow is considered *regular* if its absolute angular momentum is *negative*, and the flow is considered *anomalous* if its absolute angular momentum is *positive*.

Case 1a. $\frac{\partial \Phi}{\partial n} > 0$, $V_g > 0$, R>0: $V = -\frac{fR}{2} + \left(\frac{f^2R^2}{4} - R\frac{\partial \Phi}{\partial n} \right)^{1/2} = -\frac{fR}{2} + \left(\frac{f^2R^2}{4} + fRV_g \right)^{1/2} > -\frac{fR}{2}$.

Signs of individual terms in $-\frac{\partial \Phi}{\partial n} - fV - \frac{V^2}{R} = 0$:

$$-\frac{V^2}{R} < 0, \ -fV > 0, \ -\frac{\partial \Phi}{\partial n} < 0,$$

which means that Coriolis and pressure-gradient forces are oppositely directed, so the flow is *baric*.

From $fR < 0$ and $\frac{V_g}{V} = 1 + \frac{V}{fR}$ it follows that circulation under consideration is *anticyclonic* and $\frac{V_g}{V} < 1$.

Because $V > -\frac{fR}{2}$ and R>0, the absolute angular momentum is positive, $VR + \frac{fR^2}{2} > 0$, and the flow is thus *anomalous*.

Due to the fact that $\frac{\partial \Phi}{\partial n} > 0$ and R>0, which means that pressure grows towards the center of circulation, the flow should be labeled *anomalous high* (following terminology of the textbook) and may be regarded as the Southern Hemisphere counterpart of the Northern Hemisphere Case 2c considered in Class 40.

Because $-R\frac{\partial\Phi}{\partial n}<0$, the condition $\left|fV_g\right|=\left|\frac{\partial\Phi}{\partial n}\right|<\frac{|R|\,f^2}{4}$ must be satisfied to keep $\frac{f^2R^2}{4}-R\frac{\partial\Phi}{\partial n}$ > 0. This limits the pressure gradient and geostrophic wind speed magnitudes in high flows, both anomalous and regular (like Northern Hemisphere Cases 2c and 2d considered in Class 40).

Case 1b. $\frac{\partial\Phi}{\partial n}>0$, $V_g>0$, $R>0$: $V=-\frac{fR}{2}-\left(\frac{f^2R^2}{4}-R\frac{\partial\Phi}{\partial n}\right)^{1/2}=-\frac{fR}{2}-\left(\frac{f^2R^2}{4}+fRV_g\right)^{1/2}<-\frac{fR}{2}$.

Signs of individual terms in $-\frac{\partial\Phi}{\partial n}-fV-\frac{V^2}{R}=0$:

$$-\frac{V^2}{R}<0,\ -fV>0,\ -\frac{\partial\Phi}{\partial n}<0,$$

so the flow is *baric*.

Because $fR<0$ in $\frac{V_g}{V}=1+\frac{V}{fR}$, the flow is *anticyclonic* and $\frac{V_g}{V}<1$.

The flow should be classified as *regular* ($V<-\frac{fR}{2}$ and $R>0$, the absolute angular momentum is negative, $VR+\frac{fR^2}{2}<0$) *high* ($\frac{\partial\Phi}{\partial n}>0$, $R>0$, so the pressure increases towards the center of rotation), and may be regarded as the Southern Hemisphere counterpart of the Northern Hemisphere case 2d considered in Class 40.

Because $-R\frac{\partial\Phi}{\partial n}<0$, the condition $\left|fV_g\right|=\left|\frac{\partial\Phi}{\partial n}\right|<\frac{|R|\,f^2}{4}$ must be satisfied to keep $\frac{f^2R^2}{4}-R\frac{\partial\Phi}{\partial n}$ > 0. This limits the pressure gradient and geostrophic wind speed magnitudes in high flows, like in Case 1a above and in the Northern Hemisphere cases 2c, 2d (Class 40).

Case 1c. $\frac{\partial\Phi}{\partial n}>0$, $V_g>0$, $R<0$: $V=-\frac{fR}{2}+\left(\frac{f^2R^2}{4}-R\frac{\partial\Phi}{\partial n}\right)^{1/2}=-\frac{fR}{2}+\left(\frac{f^2R^2}{4}+fRV_g\right)^{1/2}>-\frac{fR}{2}$.

Signs of individual terms in $-\frac{\partial\Phi}{\partial n}-fV-\frac{V^2}{R}=0$:

$$-\frac{V^2}{R}>0,\ -fV>0,\ -\frac{\partial\Phi}{\partial n}<0,$$

so the flow is *baric*.

Because $fR>0$ in $\frac{V_g}{V}=1+\frac{V}{fR}$, the flow is *cyclonic* and $\frac{V_g}{V}>1$.

The flow should be classified as *regular* ($V > -\frac{fR}{2}$ and $R < 0$, so the absolute angular momentum is negative, $VR + \frac{fR^2}{2} < 0$) *low* ($\frac{\partial \Phi}{\partial n} > 0$, $R < 0$, so the pressure increases away from the center of rotation). It may thus be considered as the Southern Hemisphere counterpart of the Northern Hemisphere case 2a presented in Class 40.

Case 2a. $\frac{\partial \Phi}{\partial n} < 0$, $V_g < 0$, $R > 0$: $V = -\frac{fR}{2} + \left(\frac{f^2 R^2}{4} - R \frac{\partial \Phi}{\partial n} \right)^{1/2} = -\frac{fR}{2} + \left(\frac{f^2 R^2}{4} + fRV_g \right)^{1/2} > -\frac{fR}{2}$.

Signs of individual terms in $-\frac{\partial \Phi}{\partial n} - fV - \frac{V^2}{R} = 0$:

$$-\frac{V^2}{R} < 0, \ -fV > 0, \ -\frac{\partial \Phi}{\partial n} > 0,$$

so the flow is *antibaric*, with $V_g < 0$ and $V > 0$. Because $fR < 0$ in $\frac{V_g}{V} = 1 + \frac{V}{fR}$, the flow is *anticyclonic* and $\frac{V_g}{V} < 1$.

The flow should be classified as *anomalous* ($V > -\frac{fR}{2}$ and $R > 0$, so the absolute angular momentum is positive, $VR + \frac{fR^2}{2} > 0$) *low* ($\frac{\partial \Phi}{\partial n} < 0$, $R > 0$, so the pressure decreases towards the center of rotation), and may thus be regarded as the Southern Hemisphere counterpart of the Northern Hemisphere case 1c considered in Class 40.