Lecture 36. November 28, 2016

Topics: Solutions of the gradient wind equation. Baric and antibaric flows. Regular and anomalous flows. Atmospheric circulations associated with different gradient-wind force balances.

Reading: Holton and Hakim Section 3.2.

1. Solution of the gradient wind equation

In Class 39, we derived the gradient wind (balanced flow) equation,

$$\frac{V^2}{R} + fV = -\frac{\partial \Phi}{\partial n},$$

and obtained the following solution for the gradient wind speed:

$$V = -\frac{fR}{2} \pm \left(\frac{f^2 R^2}{4} - R\frac{\partial \Phi}{\partial n} \right)^{1/2},$$

which can be expressed in terms of geostrophic wind velocity $V_g = -\frac{1}{f}\frac{\partial \Phi}{\partial n}$ as

$$V = -\frac{fR}{2} \pm \left(\frac{f^2 R^2}{4} + fRV_g \right)^{1/2}.$$

Note that we seek V values that are real and non-negative (according to the definition of the gradient wind, see Class 39).

Consider eight possible options for gradient wind speed in the northern hemisphere ($f > 0$) with the geopotential gradient (negative of the pressure gradient force) taking on positive and negative signs.

1. Case of $\frac{\partial \Phi}{\partial n} > 0$, that is $V_g < 0$ (because $V_g = -\frac{1}{f}\frac{\partial \Phi}{\partial n}$, see Class 39).

1a. $R > 0$ and $V = -\frac{fR}{2} + \left(\frac{f^2 R^2}{4} - R\frac{\partial \Phi}{\partial n} \right)^{1/2} = -\frac{fR}{2} + \left(\frac{f^2 R^2}{4} + fRV_g \right)^{1/2}$ < 0: unphysical.

1b. $R > 0$ and $V = -\frac{fR}{2} - \left(\frac{f^2 R^2}{4} - R\frac{\partial \Phi}{\partial n} \right)^{1/2} = -\frac{fR}{2} - \left(\frac{f^2 R^2}{4} + fRV_g \right)^{1/2}$ < 0: unphysical.

1c. $R < 0$ and $V = -\frac{fR}{2} + \left(\frac{f^2 R^2}{4} - R\frac{\partial \Phi}{\partial n} \right)^{1/2} = -\frac{fR}{2} + \left(\frac{f^2 R^2}{4} + fRV_g \right)^{1/2}$ > 0: physical.

1d. $R < 0$ and $V = -\frac{fR}{2} - \left(\frac{f^2 R^2}{4} - R\frac{\partial \Phi}{\partial n} \right)^{1/2} = -\frac{fR}{2} - \left(\frac{f^2 R^2}{4} + fRV_g \right)^{1/2}$ < 0: unphysical.

2. Case of $\frac{\partial \Phi}{\partial n} < 0$, that is $V_g > 0$.

2a. $R > 0$ and $V = -\frac{fR}{2} + \left(\frac{f^2 R^2}{4} - R\frac{\partial \Phi}{\partial n} \right)^{1/2} = -\frac{fR}{2} + \left(\frac{f^2 R^2}{4} + fRV_g \right)^{1/2}$ > 0: physical.

2b. $R > 0$ and $V = -\frac{fR}{2} - \left(\frac{f^2 R^2}{4} - R\frac{\partial \Phi}{\partial n} \right)^{1/2} = -\frac{fR}{2} - \left(\frac{f^2 R^2}{4} + fRV_g \right)^{1/2}$ < 0: unphysical.

2c. $R < 0$ and $V = -\frac{fR}{2} + \left(\frac{f^2 R^2}{4} - R\frac{\partial \Phi}{\partial n} \right)^{1/2} = -\frac{fR}{2} + \left(\frac{f^2 R^2}{4} + fRV_g \right)^{1/2}$ > 0: physical.

2d. $R < 0$ and $V = -\frac{fR}{2} - \left(\frac{f^2 R^2}{4} - R\frac{\partial \Phi}{\partial n} \right)^{1/2} = -\frac{fR}{2} - \left(\frac{f^2 R^2}{4} + fRV_g \right)^{1/2}$ > 0: physical.

We are thus left with cases 1c, 2a, 2c, and 2d, which are physical (i.e., provide real and positive *V*).

2. Force balances and flow patterns corresponding to physical flow cases

Case 1c (see Fig 3.5c in the textbook for the illustration of the corresponding force balances).

$$\frac{\partial \Phi}{\partial n} > 0,\ V_g < 0,\ R < 0: \qquad V = -\frac{fR}{2} + \left(\frac{f^2 R^2}{4} - R\frac{\partial \Phi}{\partial n} \right)^{1/2} = -\frac{fR}{2} + \left(\frac{f^2 R^2}{4} + fRV_g \right)^{1/2} > -\frac{fR}{2}.$$

Signs of individual terms in $-\frac{\partial \Phi}{\partial n} - fV - \frac{V^2}{R} = 0$ (it is just another way to write $\frac{V^2}{R} + fV = -\frac{\partial \Phi}{\partial n}$) are

$$-\frac{V^2}{R} > 0,\ -fV < 0,\ -\frac{\partial \Phi}{\partial n} < 0,$$

so the Coriolis and pressure gradient force directions are the same.

Such flow is called *antibaric* (a flow is called *baric* when the Coriolis and pressure gradient forces are oppositely directed). In the considered flow case, $V_g < 0$ and $V > 0$, so geostrophic wind is not a good approximation of the gradient wind in the case of antibaric flow. One may also notice that from

$$\frac{V_g}{V} = 1 + \frac{V}{fR},$$

(see Class 39),

$$\frac{V}{fR} < -1 \text{ or } \frac{V}{|R|\,f} > 1,$$

which means that considered antibaric circulation is associated with relatively small-scale, but intense (high-wind-speed) vortices.

The considered circulation is *anticyclonic*, in the sense that $fR < 0$ (Class 39).

The flow is called *anomalous* if its absolute angular momentum (previously considered in Class 24) is negative. For a circular symmetric motion in the Earth's atmosphere, the absolute angular momentum per unit mass is given by

$$M_A = (V + \Omega_z R)R = (V + \Omega \sin\varphi R)R = VR + \frac{fR^2}{2},$$

so the flow under consideration, in which $R < 0$ and $V > -\frac{fR}{2}$ (and therefore $VR + \frac{fR^2}{2} < 0$), is anomalous. Circulations with positive absolute angular momentum are called *regular*.

In the textbook, the antibaric and anticyclonic flow under consideration is referred to as *anomalous* (since $VR + \frac{fR^2}{2} < 0$) *low* (because $\frac{\partial \Phi}{\partial n} > 0$, $R < 0$, and the pressure increases away from the center of rotation).

Case 2a (see Fig 3.5a in the textbook for the illustration of the corresponding force balances).

$$\frac{\partial \Phi}{\partial n} < 0,\ V_g > 0,\ R > 0: \qquad V = -\frac{fR}{2} + \left(\frac{f^2R^2}{4} - R\frac{\partial \Phi}{\partial n}\right)^{1/2} = -\frac{fR}{2} + \left(\frac{f^2R^2}{4} + fRV_g\right)^{1/2} > -\frac{fR}{2}.$$

Signs of individual terms in $-\frac{\partial \Phi}{\partial n} - fV - \frac{V^2}{R} = 0$ are

$$-\frac{V^2}{R} < 0,\ -fV < 0,\ -\frac{\partial \Phi}{\partial n} > 0,$$

so the flow is *baric*. In this case, both V_g and V are positive, and $\frac{V_g}{V} > 1$. Because $fR > 0$, the flow is *cyclonic*.

In the textbook, this flow is referred to as *regular* ($VR + \frac{fR^2}{2} > 0$) *low* ($\frac{\partial \Phi}{\partial n} < 0$, $R > 0$, so the pressure decreases toward the center of rotation).

Case 2c (see Fig 3.5d in the textbook for the illustration of the corresponding force balances).

$$\frac{\partial \Phi}{\partial n} < 0,\ V_g > 0,\ R < 0: \qquad V = -\frac{fR}{2} + \left(\frac{f^2R^2}{4} - R\frac{\partial \Phi}{\partial n}\right)^{1/2} = -\frac{fR}{2} + \left(\frac{f^2R^2}{4} + fRV_g\right)^{1/2} > -\frac{fR}{2}.$$

Signs of individual terms in $-\frac{\partial \Phi}{\partial n} - fV - \frac{V^2}{R} = 0$ are

$$-\frac{V^2}{R} > 0,\ -fV < 0,\ -\frac{\partial \Phi}{\partial n} > 0,$$

so the flow is *baric*. In this case, both V_g and V are positive. Because $fR < 0$ and $\frac{V_g}{V} < 1$, the flow is *anticyclonic*.

In the textbook, this flow is referred to as *anomalous* ($VR+\frac{fR^2}{2}$ < 0) *high* ($\frac{\partial\Phi}{\partial n}<0$, R < 0, and the pressure decreases away from the center of rotation).

Note that in the considered case $-R\frac{\partial\Phi}{\partial n}<0$, so in order to keep $\frac{f^2R^2}{4}-R\frac{\partial\Phi}{\partial n}$ > 0, the following condition must be satisfied: $\left|fV_g\right|=\left|\frac{\partial\Phi}{\partial n}\right|<\frac{|R|f^2}{4}$. This makes the pressure field close to the center of the high flat and winds gentle compared to their counterparts in central regions of lows.

Case 2d (see Fig 3.5b in the textbook for the illustration of the corresponding force balances).

$$\frac{\partial\Phi}{\partial n}<0,\ V_g>0,\ R<0: \qquad V=-\frac{fR}{2}-\left(\frac{f^2R^2}{4}-R\frac{\partial\Phi}{\partial n}\right)^{1/2}=-\frac{fR}{2}-\left(\frac{f^2R^2}{4}+fRV_g\right)^{1/2}<-\frac{fR}{2}.$$

Signs of individual terms in $-\frac{\partial\Phi}{\partial n}-fV-\frac{V^2}{R}=0$ are

$$-\frac{V^2}{R}>0,\ -fV<0,\ -\frac{\partial\Phi}{\partial n}>0,$$

so the flow is *baric*. In this case, both V_g and V are positive. Because $fR<0$ and $\frac{V_g}{V}<1$, the flow is *anticyclonic*.

In the textbook, this flow is referred to as *regular* ($VR+\frac{fR^2}{2}$ > 0) *high* ($\frac{\partial\Phi}{\partial n}<0$, R < 0, so the pressure decreases away from the center of rotation).

Note that in the considered case again, like in the case 2c of the anticyclonic flow, $-R\frac{\partial\Phi}{\partial n}<0$, so in order to keep $\frac{f^2R^2}{4}-R\frac{\partial\Phi}{\partial n}$ > 0, the following condition must be satisfied:

$$\left|fV_g\right|=\left|\frac{\partial\Phi}{\partial n}\right|<\frac{|R|f^2}{4}.$$

However, in this case of the regular high flow, the gradient wind speed V is smaller than in the case of anomalous high flow (case 2c) for same values of f and R.

3. Summary of terms regarding circulation types

Regular vs. anomalous. Circulation with absolute angular momentum $VR+fR^2/2$ having the same sign as planetary angular momentum $fR^2/2$ in the corresponding hemisphere, is called *regular*. Circulation with the absolute angular momentum having opposite sign to the planetary angular momentum in the corresponding hemisphere, is called *anomalous*.

Baric vs. antibaric. Circulation is called *baric* if the Coriolis and pressure gradient forces are oppositely directed. Circulation is called *antibaric* if the Coriolis and pressure gradient forces point in the same direction.

Low vs. high. Circulation is called *low* if the pressure increases away from the center of rotation. Circulation is called *high* if the pressure increases toward the center of rotation.

Cyclonic vs. anticyclonic. Circulation is called *cyclonic* if it follows direction of the Earth's rotation ($fR > 0$). Circulation is called *anticyclonic* if it is opposite to direction of the Earth's rotation ($fR < 0$).