Practical Guide to Solving Quantitative Problems

METR 3113, Atmospheric Dynamics I
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A comment frequently uttered by students in the "hard sciences" (e.g., mathematics, physics, chemistry) and engineering is

"I literally freeze when I see 'word problems' and don't even know where to begin!"

In reality, the statement is more accurately given by

"I have memorized as many formulas as my brain can hold, but I never seem to apply them correctly to the problems I'm given....or, I understand the problems described in class, but when the instructor adds a 'twist' on an exam, I go down in flames".

Someone once said that *education* is what remains when the details have been forgotten. How then does one assimilate information so that it becomes "part of them"? The key lies in <u>understanding</u> rather than <u>memorization</u>. Here's a simple test - *if you can explain something to someone else, then chances are you understand what you're talking about*.

This handout provides some very simple rules that will help you develop effective skills for solving quantitative problems. Like most other skills, <u>practice makes perfect – so don't use these occasionally but rather on a regular basis</u>. Also, don't be discouraged if you stumble during the first few attempts. Remember, the goal is to <u>develop</u> problem-solving skills <u>during</u> the course, so that by the end of the semester, you can see a vast improvement in your abilities. Let's now examine two important rules, followed by specific steps to use in solving a problem.

Rule 1. Memorization

You should memorize <u>fundamental definitions</u> from both mathematical as well as physical viewpoints (e.g., Newton's second law of motion $F = d(mV)/dt$), and from the equation you should be able to determine the correct dimensions. In most cases, the mathematical description should proceed from the physical understanding so that once the latter is mastered, the former is trivial. For example, rather than memorizing $F = d(mV)/dt$, you should understand that a body accelerates when a net force is applied, and that the rate of acceleration is inversely proportional to the mass. Once understood, the equation follows.

Rule 2. First Principles

You should memorize <u>only</u> those formulas that relate fundamental quantities or represent definitions (e.g., $u = dx/dt$) and not specific applications of them (e.g., $x = xo + ut$, as you probably were taught in basic physics). Having a firm understanding of the former, you should be able to derive the latter when needed, as shown in the simple example below.

Methodology for Solving Problems

- STEP 1. Carefully **read the problem**, perhaps several times, to gain a physical understanding of the situation. THINK carefully about the PHYSICAL nature of the problem before proceeding so that you have some idea of plausible answers. Also, DRAW PICTURES to visualize the situation.

- STEP 2. **Turn words into symbols** by writing two columns: "Given" and "Need to Find". In each, list the quantities given in the statement of the problem and those sought. If units are incompatible, employ conversions at this point.

- STEP 3. From the physical principles and equations available in your brain, **determine which formulas and relationships are appropriate for the problem**. ALSO, determine if any physical approximations are appropriate (e.g., a round Earth or gravity is constant).

- STEP 4. Proceed to **apply the equations step-by-step**, and do not leave out intermediate work. In so doing, you should remember four absolutely fundamental points:

  - ✓ Show each step of your work. This is important for several reasons. First, it will allow you to easily re-check your work and find mistakes. Second, it lets the instructor know what you've done. Finally, it greatly simplifies your review of the problem after several weeks or months, e.g., as you prepare for the final exam.

  - ✓ Write neatly. Most mistakes occur as a result of sloppy writing. In this course, you will be graded on neatness.

  - ✓ Explain what you're doing and number your equations. This may seem overly formal, but it will greatly improve your chances of solving the problem correctly. For example, you might say "We now eliminate W from the RHS using equation (2)."

  - ✓ Use units when substituting numbers for symbols. If the problem asks for a numerical result, be sure to write down the units of each number as you substitute them for symbols in the final equation.

- STEP 5. When you have an answer, **make certain the dimensions are consistent on both sides of the equal sign**. Correct dimensions do not guarantee a correct solution; however, incorrect dimensions are a sure sign that you made a mistake!

- STEP 6. **Above all, CHECK YOUR SOLUTION** by asking two questions. First, does the answer make sense from a physical point of view? Second, does the solution satisfy the appropriate equation or set of relationships? For example, if you are asked to calculate the acceleration of a baseball thrown from the ground and find it to be 10 times that of gravity, you've definitely made a mistake even if you're Roger Clemens!

Sample Problem

What constant force is required to decelerate a mass of 180 kg, traveling in a straight line, from a speed of 150 km/hr to a dead stop in 200 seconds?

STEP 1: **Draw a picture and understand the problem**

Start
150 km/hr

Stop
0 km/hr

STEP 2: Write down what you have and what you're asked to find, and check units.

Given   Need to Find

mass = 180 kg (constant)   constant force
straight path
initial speed = 150 km/hr
final speed = 0 km/hr
time of deceleration = 200 s

Before proceeding, check the consistency of units. Everything is MKS except for the speeds, and you should <u>convert them to m/s now</u>.

$$150\frac{km}{hr}\left(\frac{1hr}{3600\,s}\right)\left(\frac{1000\,m}{1\,km}\right)=41.67\,m/s$$

STEP 3: **Determine which equation to use**

At this point, you recognize that you are given velocity (actually, the change in velocity), mass, and time, and you require force. This means that you'll somehow be using Newton's second law F=ma. However, you are not given acceleration, but rather the change of velocity over time (which is in fact the acceleration in disguise)! So, you recall that F=ma is really F=m*(change in velocity/change in time). This equation therefore has everything you need! Note also that the mass is constant. If this were a car, the mass might change as the car uses gasoline.

STEP 4: Solve the problem in a step-by-step fashion (write down the explanations as shown)

Because the force is in a single direction, we can dispense with complicated vector notation and write our equation in scalar form as

$$F=\frac{d}{dt}(mV)$$

At this point, most students are tempted to make a **HUGE** misstep that **ALWAYS** should be avoided. Specifically in this example, we are given m = 180 kg, a change in velocity of 41.67 m/s, and a change in time of 200 s. This tempts one to write $\Delta V = 41.64$ and $\Delta t = 200$, from which one can **lazily** conclude (recall that mass is constant)

$$F = \frac{d}{dt}(mV) = m\frac{dV}{dt} = 180\frac{41.67}{200} = 37.50 \text{ kg m s}^{-2}.$$

YOU SHOULD NEVER EVER SUBSTITUTE FINITE NUMBERS INTO A DIFFERENTIAL EQUATION WITHOUT INTEGRATING!!!!! NOT EVER!!!!

Why? Recall from basic calculus the definition of the derivative:

$$\frac{dy}{dt} = \lim_{\Delta t \to 0}\frac{y(t+\Delta t) - y(t)}{\Delta t} = \lim_{\Delta t \to 0}\frac{\Delta y}{\Delta t}$$

The left hand side is a differential, i.e., infinitesimally small quantities, while the middle and right hand expressions consist of finite numbers in the numerator and denominator. The left becomes the middle/right only in the limit of a vanishing denominator. Consequently, you should **never** substitute finite numbers directly into a differential equation without first integrating, because the process of integration produces finite values via the limits of integration. Furthermore, failure to follow this important rule can lead to another **huge** pitfall that easily is avoided. To demonstrate the latter points as well, let's proceed to correctly solve the problem shown above by integrating Newton's second law of motion:

$$\int_{F_{initial}}^{F_{final}} F dt = \int_{(mV)_{initial}}^{(mV)_{final}} \frac{d}{dt}(mV) dt$$

where dummy variables of integration have not been used because the symbology is obvious. By performing the integration **explicitly**, we **FORCE** ourselves to be asked an important question: Can both F and m be taken outside the integrals? Were we to work the problem as shown at the top of the page, the assumption of constant mass is obvious whereas the assumption of a constant force is never explicitly used – yet it is a vital part of the problem. Based upon the information given, both F and m are constant and thus we may write

$$F\int_{t_{initial}}^{t_{final}} dt = m\int_{V_{initial}}^{V_{final}} \frac{dV}{dt} dt \quad \text{or} \quad F\int_{t_{initial}}^{t_{final}} dt = m\int_{V_{initial}}^{V_{final}} dV$$

Integrating immediately yields

$$F = m\left(\frac{V_{final} - V_{initial}}{t_{final} - t_{initial}}\right).$$

Proceeding as before, from step 2 we are given m = 180 kg, $V_{final} = 0$ m/s, $V_{initial} = 41.67$ m/s, $t_{final} = 200$ s, and $t_{initial} = 0$ s.

Substituting these values into the above equation gives

$$F = 180kg \left(\frac{0 - 41.67 m/s}{200 - 0 \sec} \right) = -37.50 \, kg \, m \, s^{-2}$$

Note that a negative force indicates *deceleration*, and thus the solution is consistent with the statement of the physical problem. Note, however, that in our hasty solution constructed above, i.e., the one in which we did not integrate, **we obtained the wrong sign**!! Thus, our use of explicit integration solved that problem and also would have been necessary in the event that the force specified was a function of time.

STEP 5: Dimensions and Units Check (don't need to write this down on your solution sheet)

The units of force are, from F=ma, kilograms times meters per second squared. This agrees with the right hand side (RHS) of our equation.

STEP 6: Does the solution make sense?

Sometimes this is a difficult question to answer. In this particular case, you may not have a feeling for what force would be required, and that's ok. As you work more and more problems, you'll develop an understanding of what numbers seem reasonable for force, acceleration, and mass. At least the minus sign in the answer was consistent with the description of the physical problem.

Final Comments

In this example, the mass was said to have been moving in a straight line. If that were not the case, you would have to consider a directional acceleration and thus work the problem in vector form. Also, you may be given pieces of information which are extraneous to the problem. For instance, you may been told in the above that the acceleration of gravity is 9.8 m s^{-2} – a true statement but meaningless for the problem! This is precisely why you should always write down what is given and what is desired. In so doing, you can more easily spot useless information that might drag you off the correct path.